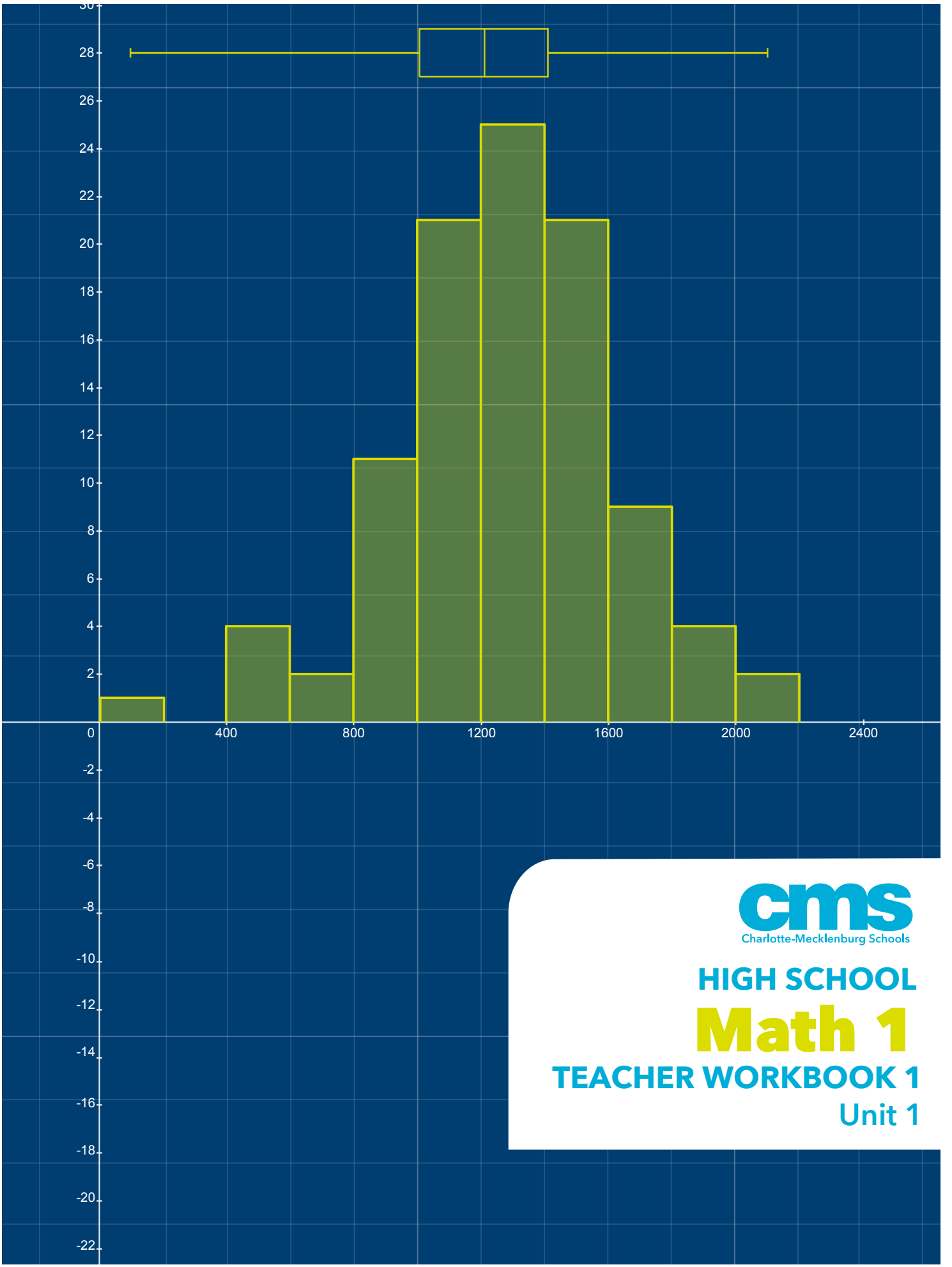


Charlotte-Mecklenburg Schools

HIGH SCHOOL
Math 1
TEACHER WORKBOOK 1
Unit 1

Table of Contents

Math 1

This set of instructional resources aims to provide a math curriculum that students and stakeholders can leverage to promote racial tolerance and oppose racism. Woven throughout the course are experiences for students and stakeholders to examine ideas of social justice, engage in current events, and expand and apply mathematics into everyday life. Reflection, student voice and agency, and high expectations are critical components of this curriculum. As a result, students consistently have opportunities to dig deeper into their worldviews and their identities as mathematicians. It is important to note that some of the topics may encourage passionate conversations and debate among students. Teachers should discuss these opportunities for student discourse during their planning meetings and recognize these possibilities when implementing the curriculum in their own classroom, leveraging their strong classroom cultures and inclusive classroom environments.

Unit 1: One-variable Statistics

The unit begins with three lessons to establish routines, build classroom culture, and gather baseline data about student knowledge. In Lessons 1–3, students will have the opportunity to learn routines such as the *Notice and Wonder* routine. They will dive into the course overview to establish and share mathematical goals of the year. In Lesson 2, students will learn how to engage in partner work through math empathy interviews and an accessible and engaging group activity. This empathy interview asks students to share with a peer about their past math experiences, as well as how they perceive themselves as mathematicians. This experience will build student understanding and expectations for what authentic group work will look and sound like throughout the year. Lastly, these lessons contain the Check Your Readiness (CYR) assessment, which is critical for teachers. The data will help teachers organize and focus the instruction and additional supports.

Unit 1 builds on the work students did in grades 6–8 using data displays (histograms, dot plots, and box plots) as a way to summarize data. These data displays are revisited in this unit but with a focus on interpretation and what they reveal about the data, in addition to the mechanics of constructing the data displays. This unit also builds on the understanding students developed around measures of center and measures of variability in grades 6–8.

Lessons 4–8 give students an opportunity to review ideas from middle school while taking the analysis of the data displays a little deeper. Lesson 4 also asks students to share their own experiences about how they travel to school and how long it takes them to get there. The lessons build on student understanding of statistical variability, descriptions of distributions, and informal comparisons of distributions. Students represent and interpret data using data displays such as dot plots, histograms, and box plots. They describe distributions using appropriate terminology such as “symmetric,” “skewed,” “bell-shaped,” “uniform,” and “bimodal.” They recognize the relationship between the mean and the median for symmetric and skewed distributions. They create data displays and calculate summary statistics using technology, then interpret the values in context.

By the end of Lesson 8, students should be able to create and interpret data displays such as histograms, dot plots, and box plots as well as calculate and interpret measures of center (mean and median) and measure of variability (interquartile range). Lessons 9 and 10 are Checkpoint Lessons. These lessons have three main purposes: 1. differentiated and small-group instruction; 2. opportunities for students to participate in various learning stations to refine and extend previous learning; and 3. the opportunity for students to complete the next unit’s CYR. Administering the CYR at this point in the unit allows plenty of time for the data to inform the next unit’s instruction. In Lessons 9 and 10, students are introduced to Desmos. Students will learn to work with technology to calculate statistics such as mean, median, quartiles, and standard deviation as well as to create data displays. The familiarity gained with technology will continue to help students throughout upcoming units as they use Desmos to explore algebraic structures and functions throughout this course.

Lessons 11–15 explore standard deviation, outliers, and comparison of data sets using measures of center and measures of variability. Lesson 14 provides an opportunity to explore the United States’ healthcare system by comparing per capita spending. Students learn that standard deviation is a measure of variability, and they interpret standard deviation in context. (Note that this unit only includes population standard deviation. Standard deviation based on a sample appears in later grades.) They understand how the presence of outliers impacts measures of center and measures of variability. Finally, students compare and interpret data sets according to their measures of center and variability, understanding that a larger standard deviation or interquartile range means more variability in the data set.

Lesson 16 occurs after administering the Unit 1 assessment and includes post-assessment activities. Taking this time to pause after the assessment to collect student reflection data through a survey and teacher conferences is a critical aspect of the course and building the classroom culture. The Student Survey is an opportunity to gather low-stakes, non-evaluative feedback for teachers to support equity and instructional pedagogy.

Instructional Routines


Aspects of Mathematical Modeling: Lessons 8, 9 & 10, 15


Card Sort: Lessons 9 & 10, 13


Co-Craft Questions (MLR5): Lessons 11, 12, 14, 15


Collect and Display (MLR2): Lessons 1, 4, 5, 6, 13, 14


Compare and Connect (MLR7): Lessons 6, 7


Discussion Supports (MLR8): Lessons 11, 13, 14


Math Talk: Lesson 13


Notice and Wonder: Lessons 1, 5, 6, 7


Round Robin: Lessons 1, 7, 15


Stronger and Clearer Each Time (MLR1): Lessons 8, 11, 12


Take Turns: Lessons 2, 7, 13, 15


Which One Doesn’t Belong?: Lessons 4, 7

Lesson 1: Welcome to Math 1!

PREPARATION

Lesson Goals	Learning Targets
• Summarize the big ideas, skills, and concepts students will explore in NC Math 1. • Communicate high expectations for all students. • Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings.	• I can articulate the big ideas we will explore in this course. • I understand the reasoning for and will strive to meet the expectations communicated by my teacher. • I know my classmates and can recognize the value I will add to this classroom community.

Lesson Narrative

This lesson marks the beginning of a rewarding journey for the class. Before jumping into the math content right away, it is important to share with students some of the major themes of Math 1, establish some agreed-upon norms, and lay the foundation for a positive classroom climate in which students feel safe to take risks, make mistakes, and grow in both their understanding of mathematics as well as their attitudes towards the field of mathematics.

What do you hope to learn about your students during this lesson? What do you hope your students will learn about you?

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
  - Name Tent feedback form (print on cardstock 1 copy per student)
  - Markers, colored pencils, etc.
- **Activity 1** (*10 minutes*)
  - Course Overview (print 1 copy per student)
- **Activity 2** (*10 minutes*)
  - "Math Is For Everybody" video by Francis Su: http://bit.ly/MathIsForEverybody
- **Activity 3** (*10 minutes*)
  - Course Syllabus (teacher supplied - print 1 per student)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)

LESSON

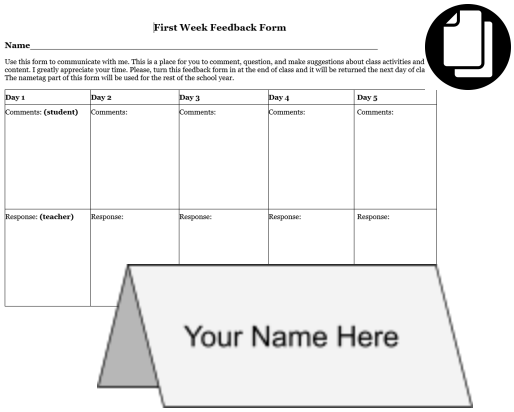
Warm-up: Name Tent (*5 minutes*)

Students will begin the lesson by creating a name tent.[1] The name tent makes learning students' names easier and provides space for them to share their thoughts and questions at the end of each class period for the first five days of class. This activity facilitates building a partnership between students and teacher. It may be beneficial to ask students to write their preferred name and/or pronouns on or in the name tent, as well.

Student Task Statement

Welcome to Math 1!

1. As you enter the classroom, locate your assigned seat.
2. On your desk, you will find a printout titled “First Week Feedback Form.” Please write your name on the Name line and then fold the paper in half so that the printed side is inside the fold to make a tent.
3. On the outside of the tent, please write your name on both sides (large enough for everyone to be able to read as needed).
4. Introduce yourself to the classmates sitting closest to you.
5. If you have time, you may decorate the outside of your name tent.



Activity 1: What Is Math 1? (*10 minutes*)

Instructional Routine: Notice and Wonder

The purpose of this activity is to provide students an opportunity to see what topics are included in Math 1 and informally share with the class what topics they may already have some experience with from earlier courses or from their own life experiences.

This is the first time in the course that students will participate in a *Notice and Wonder* routine.

NOTICE AND WONDER

What Is This Routine? Students are shown some media or a mathematical representation. The prompt to students is “What do you notice? What do you wonder?” Students are given a few minutes to think of things they notice and things they wonder, and share them with a partner. Then, the teacher asks several students to share things they noticed and things they wondered; these are recorded by the teacher for all to see. Sometimes the teacher steers the conversation to wondering about something mathematical that the class is about to focus on.

Why This Routine? The purpose of the *Notice and Wonder* routine is to make a mathematical task accessible to all students with these two approachable questions. By thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1).

[1] Adapted from Sara VanDerWerf

Step 1

- Share the Math 1 Course Overview with students. Instruct them to briefly skim the topics included in the Math 1 course and use the graphic organizer in their student workbook to record at least one thing they notice and one thing they wonder about Math 1.
- After 2 minutes, have students turn to a partner and share their notice/wonder with one another.

Step 2

- Ask students to share the things they noticed and wondered. Record and display several responses for all to see. It is important to capture student responses as authentically as possible, without commentary or edits. This validates student thinking and sense making. After recording all shared responses, ask students, "Is there anything on this list that is creating additional noticings or wonderings for you?"
- As students share, listen for and elevate connections they make between prior math courses and topics in Math 1. Also highlight some of the real-world applications students anticipate exploring.

Student Task Statement

What am I going to learn and do in Math 1?

1. Briefly skim the Math 1 course overview provided by your teacher.
2. Use the graphic organizer below to record at least one thing you notice and one thing you wonder about Math 1. You can certainly list more than one!

I Noticed...	I Wondered...

3. Be ready to share what you noticed and wondered with your class.

Activity 2: What Does It Mean To Be a Mathematician in This Class? (*10 minutes*)

Instructional Routines: Round Robin; Collect and Display (MLR2)

The purpose of this activity is to encourage students to articulate for themselves the mindset and habits that support learning in mathematics. In this activity, students respond to a short video that confronts the notion that only certain people are “math people.” They work towards an agreed-upon set of habits and mindsets that mathematicians work within, and they apply this definition to describe what it means to be a mathematician as a student in this course.

This is the first time in the course that students will participate in a *Round Robin* routine and a *Collect and Display* routine.

ROUND ROBIN	***What Is This Routine?*** In small groups, students take turns sharing their rough draft response to an open-ended question while other group members refrain from comments or questions. A prop can be passed within each group to indicate whose turn it is to talk. After each student has had a turn to share, the group can ask questions of each other; then the teacher selects students to share with the whole class what their group members said.

	Why This Routine? Engaging a group of students in collaborative problem solving, with equitable inclusion of ideas, can be challenging due to normative social status issues that place higher value on some students' contributions over others. *Round Robin* allows all students to include their rough draft ideas for solving an open-ended problem without a subset of students dominating the conversation. Knowing all ideas will be shared should motivate all students to try at least one strategy to solve a problem on their own, critical for making sense of problems and persevering in solving them (MP1). The active sharing and listening involved in this routine also provides opportunity for constructing and critiquing viable arguments (MP3).

COLLECT AND DISPLAY 	***What Is This Routine?*** The teacher captures students' oral words and phrases into a stable, collective reference in order to stabilize the fleeting language that students use during partner, small-group, or whole-class activities. The teacher listens for, and scribes, the student output using written words, diagrams, and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use over the course of a lesson or unit. ***Why This Routine?*** *Collect and Display* (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students' own output to be used as a reference in developing their mathematical language over time.

Step 1

- Play the video, "Math is for Everybody" by Francis Su, for the whole class.

Step 2

- Ask students to arrange themselves in small groups or use visibly random grouping. Provide groups with 6 minutes to take turns discussing the prompts using the *Round Robin* routine.
- As students discuss, use the *Collect and Display* routine: listen to the language students use with each other, and capture their oral words and phrases on the board or on chart paper so that students can refer back to the language they used in this activity during Activity 3.

Student Task Statement

In your small group, reflect on the video, "Math Is for Everybody" by Francis Su, by discussing the following questions with your group:

- Do you believe that some people are "math people" and others are not?
- What makes someone a mathematician?
- In what ways can someone work to get better at a sport, playing a musical instrument, or some other hobby?
- How can a student who believes math is for everybody approach this course?

Activity 3: What Does My Teacher Expect From Me? (*10 minutes*)

The purpose of this activity is to briefly introduce students to some of the classroom policies and procedures. Keep in mind that it is not best practice to read the entire syllabus directly to students. Additionally, there will be plenty of opportunities within the coming days to walk students through certain classroom routines as they naturally arise in daily activities. Plan ahead by re-reading the syllabus and noting which expectations and procedures students need to know right now. Setting a timer for this activity will help the discussion stay focused and concise.

Step 1

- Provide students with the course syllabus. Provide 3 minutes for students to read it quietly to themselves.

Step 2

- Highlight for students the expectations and procedures that they need to know right now. Pay special attention to any expectations that connect to the thoughts collected from small groups in Activity 2. Remind them that they will be supported in learning all of the expectations and procedures as they naturally arise in the course. Provide time for students to ask any questions they have at this point.

Student Task Statement

Briefly review the course syllabus. Feel free to highlight and annotate your copy.

Lesson Debrief (*5 minutes*)

In this lesson, students learned what big topics they will explore in Math 1. They also were introduced to the classroom expectations and reflected on what it means to be a mathematician in Math 1. Provide some quiet time for students to create their own vision of themselves in this class.

Student Reflection

What are you most excited about in this class? Write one or two sentences OR draw a picture representing your response.

Cool-down: Student-to-Teacher Feedback (*5 minutes*)

Students leave feedback for the first column inside their Name Tent and turn it in if this is the end of the class period. This is an excellent opportunity to model where in the classroom future work is to be turned in.

Cool-down

Open your Name Tent. Write a note in the top left box by commenting, asking a question, or making a suggestion about class activities and content.

First Week Feedback Form

Name_____

Use this form to communicate with me. This is a place for you to comment, question, and make suggestions about class activities and content. I greatly appreciate your time. Please, turn this feedback form in at the end of class and it will be returned the next day of class. The nametag part of this form will be used for the rest of the school year.

Day 1	Day 2	Day 3	Day 4	Day 5
Comments: **(student)**	Comments:	Comments:	Comments:	Comments:

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What do your students think it means to be good at math? How are you helping them change negative impressions they might have about their ability to reason mathematically?

Lesson 2: Building a Math Community

PREPARATION

Lesson Goals	Learning Targets
• Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings. • Recognize the value that making mistakes and engaging in productive struggle bring to learning mathematics.	• I know my classmates and can recognize the value I will add to this classroom community. • I understand that making mistakes and engaging in a productive struggle are necessary steps in growing as a mathematician.

Lesson Narrative

In this lesson, students are introduced to the reasoning behind Check Your Readiness activities and participate in the first Check Your Readiness set of questions. The major activity, a Math Empathy Interview, encourages students to share their personal experiences in math classes in the past with a peer. Students should walk away from this lesson understanding that making mistakes and engaging in productive struggle are both necessary steps in growing as a mathematician.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Agenda, Materials, and Preparation

- **Warm-up** (*10 minutes*)
  - Unit 1 Check Your Readiness (Part One) (print 1 copy per student)
- **Activity 1** (*20 minutes*)
  - Popsicle sticks or strips of paper with the name of half of your class roster placed in a cup
- **Lesson Debrief** (*10 minutes*)
  - "Mistakes Are Powerful" video: http://bit.ly/MistakesArePowerful
- **Cool-down** (*5 minutes*)
  - Place Name Tents on students' desks prior to the beginning of class.

LESSON

Warm-up: Unit 1 Check Your Readiness (Part One) (*10 minutes*)

Check Your Readiness (CYR) questions are a valuable tool in Math 1. While the lesson activities in Math 1 are designed to activate prior learning to scaffold students up to the state standards, sometimes students need more explicit support on prerequisite concepts. Therefore, additional supports have been embedded where appropriate. Questions within these CYR activities can inform data-based decision-making when activities are provided that are optional, such as the Bridge tasks. They can also be used to determine selection of Practice Problems and decisions for what students experience in the Checkpoint lessons.

Before beginning the CYR, have students read the opening explanation and directions, and answer any questions they may have. After students have read the explanation for CYR, reiterate to students the purpose of these activities, to ensure all students are getting the instruction they need. Also, reassure students that this is not a test but rather an opportunity to demonstrate what they already know as well as the areas where they need help. Let them know the goal is to make sure every student has a positive and successful year in mathematics that includes learning lots of math, honing their skills in communicating mathematics, and having fun.

- Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Activity 1: Math Empathy Interview (*20 minutes*)

Instructional Routine: Take Turns

The purpose of this activity is to develop connections and relationships with peers. Through this activity, students will discover that all students experience struggle at some point and that there are different ways to respond to that struggle, some ways are more productive than others. As a result of this activity, students gain empathy for one another that will provide a safe classroom community where mistakes and struggle are considered valuable to the learning process.

This is the first time in the course that students will participate in a *Take Turns* routine.

TAKE TURNS

What Is This Routine? Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner's arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

Prior to the lesson, place popsicle sticks or slips of paper with student names for one half of the class roster in a cup.

Step 1

- Display the Math Empathy Interview Protocol on the board for all to see.
- Review the goal, norms, and high-level overview of the protocol.
- Ask students: "Can someone restate what we will be doing in these interviews?"
- Walk around to each student not represented in the cup with student names and ask them to draw one of the popsicle sticks or slips of paper. The person they select will be their partner for the interview. When all popsicle sticks/slips of paper have been drawn, instruct students to locate their interview partner (scanning the Name Tents) and relocate to a desk next to that student.

Step 2

- Students have 12 minutes to conduct the interviews (6 minutes per interview).
- At the conclusion of the interviews, students reflect on three things that stood out to them about the interview.

Student Task Statement

Math Empathy Interview Protocol

Goals: To gain a deeper understanding of another student's experience in math

Norms for the interview:

- Interviewee:
  - Share only what you feel comfortable sharing
- Interviewer:
  - Seek to understand, not confirm (for example, do not agree or disagree with statements given)
  - Ask once, clearly
  - Probe: tell me more… what was that like for you

During an empathy interview, the interviewer asks the questions and the interviewee answers them. The interviewer can only ask probing questions, not share their own experiences.

Instead of writing down responses to each of the questions below, listen intently for big ideas so that you can capture three things that stood out to you.

After 6 minutes, the interviewer and interviewee switch roles.

Empathy Interview Questions:

1. Tell me about a time where you felt successful in math.
   - Why do you think you were successful?
   - What did you do?
   - What did others do (your classmates, teacher, friends)?
2. Tell me about a time where math was hard.
   - How did it feel?
   - What did you do?
   - Why was it hard?
   - What do you wish would have happened?
   - What would have helped?
3. What comes to mind when I say, "There is no such thing as a math person"?
4. Describe or draw a picture on a whiteboard or scrap piece of paper of what happens in your brain when you see a math problem you don't understand.
5. What advice would you give another student who thinks they aren't good at math?
6. Draw a picture on a whiteboard or scrap piece of paper of what you think about when you hear "growth mindset in math."

The mathematician I interviewed was:	Three things that stood out to me from the interview were:
	1. 2. 3.

Lesson Debrief (*10 minutes*)

The goal of this lesson was to help students recognize that struggle is a normal part of learning mathematics and that they are not alone in experiencing struggle.

Play the 3-minute video, "Mistakes are Powerful."

To synthesize the goal of the lesson for students, ask the following questions:

- "Raise your hand if, during the interview, you discovered something you and your interview partner have in common."
  - Select a student with a raised hand and ask, "What did you discover you and your partner have in common?" (Sample response: That we both have made mistakes or struggled in math class before.)
- "What do you think is the difference between struggle and productive struggle?" (Sample response: Productive struggle sounds like something is hard but you are making progress and working through stuff. Struggle sounds like it is just hard and you don't know what to do next.)

PLANNING NOTES

Student Lesson Summary and Glossary

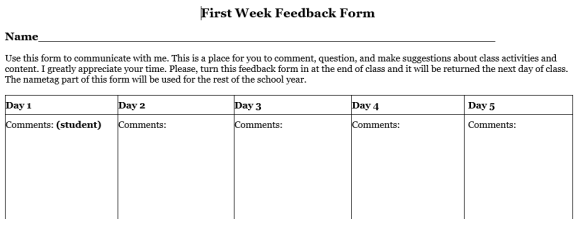
All students make mistakes and experience struggle at some point in learning mathematics. There are different ways to respond to that struggle, and some ways are more productive than others. Making mistakes and engaging in productive struggle are valuable parts of learning mathematics, and both provide a strong foundation to build a safe classroom community.

Cool-down: Student-to-Teacher Feedback (*5 minutes*)

Students leave feedback for the second column inside their Name Tent and turn it in if this is the end of the class period. This is an excellent opportunity to model where in the classroom future work is to be turned in. Continue to use these Name Tents and gather student feedback and questions over the remainder of the week.

Cool-down

Open your Name Tent. Write a note in the appropriate box in the top row of the table by commenting, asking a question, or making a suggestion about class activities and content.



First Week Feedback Form

Name_____

Use this form to communicate with me. This is a place for you to comment, question, and make suggestions about class activities and content. I greatly appreciate your time. Please, turn this feedback form in at the end of class and it will be returned the next day of class. The nametag part of this form will be used for the rest of the school year.

Day 1	Day 2	Day 3	Day 4	Day 5
Comments: **(student)**	Comments:	Comments:	Comments:	Comments:

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Think about a time you made a mistake during math class. How did you leverage your mistake to show students that mistakes are just learning in process?

Lesson 3: Great Group Work

PREPARATION

Lesson Goals	Learning Targets
• Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings. • Articulate characteristics of great group work.	• I know my classmates and can recognize the value I will add to this classroom community. • I can articulate what great group work looks like.

Lesson Narrative

The purpose of this lesson is to further build a strong math community in the classroom while also facilitating the establishment of expectations around collaboration and group work.

What does productive group work look like? What are the benefits of investing time to establish group work expectations early on?

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
  - Unit 1 Check Your Readiness (Part Two) (print 1 copy per student)
- **Activity 1** (*25 minutes*)
  - 4 different color highlighters per group of 4 students
  - 1–100 Numbers to Get Students Talking (print 3 copies per group of 4 students)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*10 minutes*)
  - Place Name Tents on students' desks prior to the beginning of class.
  - M1.U1.L3 Cool-down (print 1 copy per student)

LESSON

Warm-up: Check Your Readiness (Part Two) (*5 minutes*)

- Administer Part Two of the Unit 1 Check Your Readiness.
- Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Activity 1: 100 Numbers to Get Students Talking[1] (*25 minutes*)

The purpose of the 100 Numbers to Get Students Talking activity is to establish classroom norms around group work. Group work will be a regular occurrence in this course, and it is important that students have a clear idea of what is expected.

Step 1

- Ask students to arrange themselves in groups of four or or use visibly random grouping(preferably a different group than in previous lessons so students are meeting new classmates).
- Tell students the expectation in this class over the entire course is to talk about math out loud every day. Most lessons will include a partner or group activity that includes discussion. Today's class will include practice working in groups.

Step 2

- Give directions for the task.
  - Each group will have 3 minutes to find as many numbers from 1–100 in order.
  - Group members will take turns going in a circle, highlighting each number as they find it.
    - Student 1 highlights the number 1.
    - Student 2 highlights 2.
    - Student 3 highlights 3.
    - Student 4 highlights 4.
  - Continue the cycle with student 1 highlighting 5, and student 2 highlighting 6, etc., finding as many numbers as possible in the provided time.
  - Students can help each other find numbers, but each student has to highlight the number assigned to them.

- Ask students to clear the tables of everything.
- Hand out one highlighter per student (a different color to each student in the group).
- Hand out one copy of the numbers 1–100 upside down to each group.

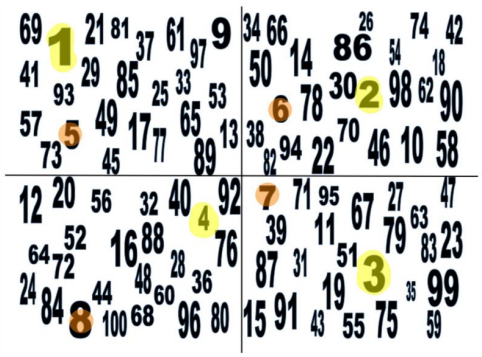
Step 3

- Start a 3-minute timer that all students can see, and say, "go." While students are working, take pictures of the groups during work time. Note: ask permission before taking pictures if student faces are visible.
- At 3 minutes, say, "hands up." Ask groups to turn over their paper and write on the back the last number that their group was able to find. (It is typical for groups to find 20–40 numbers the first time they do this activity.) Tell the class they will be doing this activity again. Have each group discuss what went well in round 1 and a strategy to do better in the next round. Give groups 2–3 minutes to discuss. Have a few groups share out what went well in round 1. Tell students to prepare for round 2 and try to find even more numbers.

[1] Adapted from Sara VanDerWerf

Step 4

- Repeat the process for round 2 using a new handout. Continue to take pictures of the groups working.
- Compare the amount of numbers found in round 1 to the amount of numbers found in round 2. Usually, all groups will have highlighted more numbers the second time. Tell students there is a pattern to the numbers. Ask students to share what they have noticed so far. Record the findings on the board. A common noticing is the first number was in the top left quadrant and the second number was in the upper right quadrant as shown (display this image after a group names it). If time allows, repeat for one additional third round.



Step 5

- Show students the photos of them working in groups.
- Ask students what they see in the pictures. What are some of the ways students are engaging? (students are leaning in, students are focused on the paper, their bodies are facing each other)
- Name that the expectation during group work is to always be so focused on the task and the group discussion that what is going on outside of the group is barely noticeable.

If possible, have the photos printed and displayed creatively in the classroom. Refer back to these photos when students need a reminder at any point in the course of what group work should look like.

Student Task Statement

Preparation:

- Arrange desks to accommodate a group of four students.
- Clear tables of everything.
- Each person in the group needs a different colored highlighter.
- Each group needs one copy of the 1–100 number sheet and must keep it UPSIDE DOWN until the teacher says, “Go.”

Directions:

Your group will have 3 minutes to find as many numbers from 1–100 in order. Group members will take turns going in a circle highlighting each number as they find it.

- Student 1 highlights 1.
- Student 2 highlights 2.
- Student 3 highlights 3.
- Student 4 highlights 4.
- When it is student 1’s turn again, highlight the next number.
- Keep the cycle going, finding as many numbers as possible in the provided time.

All group members can assist in finding the numbers, but each student has to highlight the number assigned.

Lesson Debrief (*5 minutes*)

Display the following prompt on the board for all to see: "What does great group work in math look like?" Have students journal initial thoughts on a scrap sheet of paper. After 1 minute, have students share responses in small groups and combine the responses into one group response. Then, have groups share combined ideas with the whole class, recording the ideas on the board for all to see and note any common ideas. From this list of ideas, generate a set of agreed upon ideas of what great group work in math looks like and have students write this down in the Lesson Debrief section of their workbooks.

Student Reflection

Great group work in math looks like …

_____

Cool-down: Giving Feedback (*10 minutes*)

Addressing: NC.6.SP.3.

Cool-down Guidance: This cool-down will offer students the opportunity to share how they prefer to hear feedback. It also provides some data on how students are thinking about measures of center.

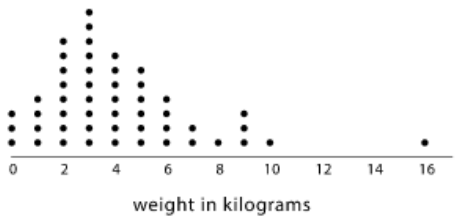
Cool-down

Below is a math problem presented to a Math 1 class along with three of the students' responses.[2]

Provide feedback for each student. Offer feedback in the manner you wish to receive feedback yourself.

This dot plot shows the weights of backpacks, in kilograms, of 50 Math 1 students at a school in North Carolina.

How would you describe a typical backpack weight at this school? Explain your thinking.



Clare's Response:

I think a typical weight is 3 because it has the most dots.

Jada's Response:

4.14

Elena's Response:

I don't remember how to solve this. I can tell that most students' backpacks weigh 2–5 pounds, but I'm not sure what the typical weight would be.

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What are some of your beliefs around students working in pairs or small groups? How will you encourage students to work together regularly throughout the course?

Lesson 4: Getting to Know You

PREPARATION

Lesson Goals	Learning Targets
• Describe (orally and in writing) the difference between statistical and non-statistical questions. • Describe (orally and in writing) the difference between numerical and categorical data.	• I can tell statistical questions from non-statistical questions and can explain the difference. • I can tell the difference between numerical and categorical data.

Lesson Narrative

The mathematical purposes of this lesson are to understand what makes a question statistical and classify data as numerical or categorical. Additionally, students continue to build community by learning more about one another through a series of survey questions. This unit begins with creating data displays and describing distributions of numerical data. **Numerical data** are responses to questions that are numbers that can be ordered in a natural way. **Categorical data** are responses to questions that fit into distinct categories.

Students learn to recognize **statistical questions** as questions that anticipate variability in the data. In this lesson, students recall the concept of variability to discuss the difference between statistical and **non-statistical questions** while they collect survey data from their classmates. (The data will be used again in later lessons, so they should be kept in a spreadsheet or a folder.) Students classify questions as being statistical or non-statistical, and classify the data that they collect from statistical questions as numerical or categorical.

When students identify data resulting from a statistical question as numerical or categorical, students are engaging in MP2 because they are having to make sense of data in relation to the question being asked. Students also build from informal to more precise language, MP6, for several vocabulary terms that will be used throughout the unit.

What about this topic brings you the most excitement or curiosity?

Focus and Coherence

Building On	Building Towards
NC.6.SP.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	**NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots).

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*20 minutes*)
  - Representing Data About You and Your Classmates question sets (print 3 copies per group of 4 students)
- **Lesson Debrief** (*10 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L4 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.SP.1

This bridge serves two purposes for students: to practice collecting data and to recall that not all statistical questions yield the same type of answers. Here, students get to practice making an informal distinction between categorical data and numerical data. This practice will be useful later in this lesson when students collect data from their classmates and try to make generalizations about the questions they ask by looking for commonalities between the type of data they collect.

Student Task Statement

Here are some survey questions. After you answer the survey questions, record answers from one other classmate.

1. How many hours of sleep do you typically get on a school night? Answer to the nearest hour.

2. How do you travel to school on most days? Choose one.
   - Walk
   - Bike
   - Scooter or skateboard
   - Car
   - School bus
   - Public transport
   - Other

3. What is the difference between the answers to question 1 and question 2?

DO THE MATH

PLANNING NOTES

Warm-up: Types of Data (*5 minutes*)

Instructional Routine: Which One Doesn't Belong?	
Building On: 6.SP.1	**Building Towards:** NC.M1.S-ID.1

This warm-up prompts students to compare four survey questions. It gives students a reason to use language precisely (MP6) and gives you the opportunity to hear how they use terminology and talk about characteristics of the items in comparison to one another.

This is the first time in the course that students will participate in a *Which One Doesn't Belong?* routine.

WHICH ONE DOESN'T BELONG?	***What Is This Routine?*** Students are presented with four figures, diagrams, graphs, or expressions with the prompt: "Which one doesn't belong?" Typically, each of the four options "doesn't belong" for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn't belong and given opportunities to make their rationale more precise. ***Why This Routine?*** *Which One Doesn't Belong?* fosters a need to define terms carefully and use words precisely (MP6) in order to compare and contrast a group of mathematical objects or representations. Because there are no wrong answers, the focus is on student reasoning, and especially on students communicating their reasoning. This routine cultivates an inclusive classroom culture by prompting students to be creative thinkers, clear communicators, and good listeners.

Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Display the survey questions for all to see, and ask students, "Which one doesn't belong?"
- Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular item does not belong and together find at least one reason each item doesn't belong.

Survey Questions

Which one doesn't belong? Explain your reasoning.

a. How many potato chips are in this bag of chips?	b. What is the typical number of chips in a bag of chips?
c. What type of chips are these?	d. What type of chips do students in this class prefer?

Step 2

- Ask each group to share one reason why a particular item does not belong.
- Record and display the responses for all to see.
- After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as numerical data, categorical data, or average. Also, press students on unsubstantiated claims.

DO THE MATH

PLANNING NOTES

Activity 1: Representing Data About You and Your Classmates (*20 minutes*)

Instructional Routine: Collect and Display (MLR2)
Building Towards: NC.M1.S-ID.1

In this activity, each group of students is assigned three questions. One of the three they are assigned is a non-statistical question; one would generate numerical data, and one would generate categorical data. Groups also generate a fourth question of their own that can be answered with data. First, the group comes up with four survey questions that they can ask their classmates to collect data about their four questions of interest. Then, they collect data from their classmates by asking the survey questions. Finally, they summarize their results to answer the four questions of interest and reflect on the nature of the different questions they attempted to answer.

At the end of the Representing Data About You and Your Classmates activity, students will need to keep their data for use in a later lesson where students will represent the data collected in this activity graphically. If they record the data in their workbooks, the data will be easy to retrieve later. If students record data some other way, be sure your method allows them to easily retrieve the data later.

Step 1

- Ask students to arrange themselves in groups of four or use visibly random grouping. Assign each group one of the following sets of three questions from the Representing Data About You and Your Classmates question sets blackline master.

Set A

1. On average, how many letters are in the family (last) names for students in this class?
2. How do students in this class like to participate in Math class? group work, individual assignments, computer activities, or a partner project?
3. In what year was the 13th Amendment ratified?

RESPONSIVE STRATEGY

Read all questions aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language

Set B

1. On average, how many social media accounts do students in this class have?
2. Which sport do students in this class most like to watch? basketball, baseball, football, or soccer?
3. Who was the first female mathematician to win a Fields medal?

Set C

1. How long did it take students in this class to get to school this morning?
2. Do students in this class believe that making mistakes in Math is a learning experience?
3. How many desks are in the classroom?

Set D

1. On average, how many movies did each student in the class watch this summer?
2. Which new pet would students in this class like the most? cat, dog, hamster, or iguana?
3. Which NFL team did mathematician John Urschel play for?

- Give groups 2 minutes to write an additional question of their own that could be answered using data from the class so that each group has four questions.
- As a whole-class discussion, share with students an example of the types of questions they should be asking their classmates. “For example, if you are assigned the question, ‘Which month has the most birthdays from the class?’ you might ask your classmates, ‘In what month is your birthday?’”

Step 2

- In order for each group to get 15 total responses for all four questions, each member of a group should ask one of the four questions to 15 classmates. Students responding to question 3 may look up the answers, if necessary. When the group comes back together, instruct students to share the responses with one another so that each person has a complete table of responses.

Step 3

- As groups work, use the *Collect and Display* routine while circulating and listening to students talk about the similarities and differences between the types of data collected. Write down common or important phrases you hear students say about each type onto a visual display (e.g., “these are all numbers” or “this only has one answer”). Collect the responses into a visual display. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed.

Why This Routine? *Collect and Display* (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students’ own output to be used as a reference in developing their mathematical language over time.

Advancing Student Thinking: Students may confuse statistical questions with survey questions. Explain that the set of three questions are statistical questions that can be answered using the survey questions. For example, students may think they should ask each of their classmates for the average distance they have traveled from home. However, students can ask each classmate "What is the furthest distance you have traveled from home?" and collectively use the answers to this survey question to answer the statistical question about the average distance their classmates have travelled from home.

Student Task Statement

Your teacher will assign your group a set of three questions.

1. As a group, write another question (#4) that will require data collected from the class to answer.
2. For each of the four questions, write a survey question that will help your group collect data from the class that can be analyzed to answer the questions.
3. Assign one survey question to each member of your group. Ask your one survey question to 15 classmates. Record classmates’ responses below.

	Classmate’s name	Survey question response
1		
2		
3		
4		

4. After collecting the data, return to your group, and together summarize the data for each question in a sentence or two.
5. With your group, decide what the responses for survey question number 1 have in common. Then do the same for survey questions number 2 and number 3.
6. Does the question you wrote (#4) fit best with the questions from numbers 1, 2, or 3? Explain your reasoning.

DO THE MATH

PLANNING NOTES

Are You Ready For More?

1. Find a news article that uses numerical data to discuss a statistical question.
2. Find a news article that uses categorical data to discuss a statistical question.

Lesson Debrief (*10 minutes*)

Share all of the questions numbered 1 from each set. Ask students to summarize what responses to these questions might have in common. (In order to answer them, you collect responses that are numbers.)

Repeat for the questions from numbers 2 and 3. (In order to answer questions from number 2, you collect responses that are descriptive words or characteristics. The questions from number 3 only have one possible answer, and don't have any variability in the responses.)

RESPONSIVE STRATEGY

Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: numerical data, categorical data, statistical questions, non-statistical questions.

Supports accessibility for: Memory; Language

Tell students that we call data collected by questions in number 1 "numerical data," data collected by questions in number 2 "categorical data," and the questions in number 3 are "non-statistical questions" because there will be no variability in the responses. Questions in numbers 1 and 2 are called "statistical questions" since they require collection of data and there is anticipated variability in the responses.

In addition to the questions above, choose whether students should first have an opportunity to reflect on the following questions in their workbooks or talk through them with a partner. Determine what questions will be prioritized in the full class discussion.

To promote student understanding of the differences between statistical and non-statistical questions and classifying data as numerical or categorical, ask:

- "What makes a question statistical?" (There is variability in the data collected.)

PLANNING NOTES

- "What is an example of a non-statistical question?" (What value for x makes the equation $x+5=7$ true?)
- "What is an example of a statistical question that we have not used in class?" (On average, how many people eat breakfast every day?)
- "What type of data is collected to answer the statistical question, 'Would the class rather have pizza or donuts?'" (Categorical)
- "What is an example of a statistical question that results in numerical data?" (What is the typical surface area of styrofoam pellets?)

To help prepare students for the next lesson, ask:

- "What are some different ways to represent data graphically?" (Bar graphs, dot plots, box plots, pie charts, and histograms.)

Student Lesson Summary and Glossary

Statistics is about using data to solve problems or make decisions. The question that you ask determines the type of data that you collect and whether or not there is variability in the data collected.

There are two types of data:

Numerical data: Data where the values are numbers, measurements, or quantities; also called measurement or quantitative data. For example, the weights of 10 different dogs are numerical data.

Categorical data: Data where the values are categories. For example, the breeds of 10 different dogs are categorical data.

In earlier grades, you learned that there is variability in a data set if not all of the values in the data set are the same. In this lesson you learned about two specific types of questions.

Statistical question: A question that can only be answered by collecting data and there is an expectation for the data to have variability.

Here are some examples of statistical questions:

- "What is the average class size at this school?" would produce numerical data with some variability.
- "What are the favorite colors of students in this class?" would produce categorical data with some variability.

Non-statistical question: A question that can be answered by a specific measurement or procedure where no variability is anticipated.

Here are some examples of non-statistical questions:

- "How many students are on the roster for this class?" would produce numerical data that do not vary. There is only one value in the data set, so there is no variability.
- "What shape is this table?" would produce categorical data that do not vary. There is only one value in the data set, so there is no variability.

Cool-down: Categorizing Questions (*5 minutes*)

Building Towards: NC.M1.S-ID.1

Cool-down Guidance: More Chances
Students will have more opportunities to explore these ideas. If students struggle with what qualifies as a statistical question, highlight the distinction again when students use the data they collected in Lesson 3.

Cool-down

Categorize each of the questions below as one of these types, then explain your reasoning for putting the question in that category.

Category types:

- **Statistical question** requiring numerical data to answer it
- **Statistical question** requiring categorical data to answer it
- **Non-statistical question**

	Category types	Explain your reasoning
1. On average, how many books does each person in the United States read each year?		
2. How many acts are in the play *Romeo and Juliet*?		
3. Which book was read most by students in the class this summer?		
4. How many books are in the classroom right now?		

Student Reflection:

I love math most when _____ and I do not like it when _____.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In future lessons, students will continue to work with the four categories they explored today (numerical data, categorical data, non-statistical questions, and statistical questions.) How does the work from today, having students generate their own questions, help students build conceptual understanding of this vocabulary?

Practice Problems

1. Write a survey question for which you would expect to collect numerical data.
2. Write a survey question for which you would expect to collect categorical data.
3. Select **all** the statistical questions.
    a. What is the typical amount of rainfall for the month of June in the Galapagos Islands?
    b. How much did it rain yesterday at the Mexico City International Airport?
    c. Why do you like to listen to music?
    d. How many songs do students in the class usually listen to each day?
    e. How many songs did you listen to today?
    f. What is the capital of Canada?
    g. How long does it typically take for 2nd graders to walk a lap around the track?

4. Read each question. Think about the data you might collect to answer it and whether you expect to see variability in the data. Complete each blank with "Yes" or "No." If you decide a question is statistical, determine if its answers will give numerical data or categorical data. If you decide a question is not statistical, then skip the question about the type of data.[1]

   a. How many cups of water do my classmates drink each day?

      - Is variability expected in the data?____
      - Is the question statistical?____
      - Will the answers give numerical data or categorical data?___

   b. Where in town does our math teacher live?

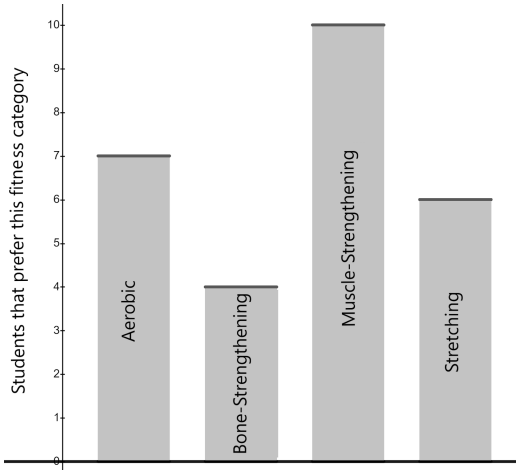
      - Is variability expected in the data?____
      - Is the question statistical?____
      - Will the answers give numerical data or categorical data?___

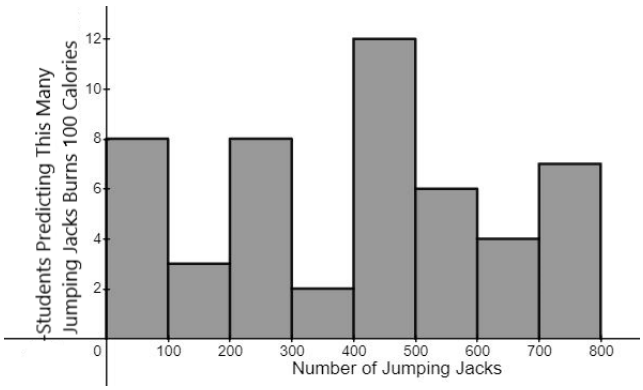
   c. How many minutes does it take students in my class to get ready for school in the morning?

      - Is variability expected in the data?____
      - Is the question statistical?____
      - Will the answers give numerical data or categorical data?___

   (Addressing NC.6.SP.1)

5. Examine the bar graph and histogram below.





   a. What differences do you notice in each representation?

   b. What question could have been asked to give the answers that are represented in the bar graph?

   c. What question could have been asked to give the answers that are represented in the histogram?

   d. How do the questions you came up with differ from one another?

   (Addressing NC.6.SP.4)

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 5: Data Representations

PREPARATION

Lesson Goals	Learning Targets
• Create a histogram and box plot to represent numerical data. • Identify (in writing) the five-number summary that describes given statistical data. • Interpret a box plot that represents a data set.	• I can find the five-number summary for data. • I can use a histogram or box plot to represent data.

Lesson Narrative

This lesson revisits prior-grade-level content. If the Check Your Readiness diagnostic questions 1 through 4 indicate that your students know the representations well, this lesson may be safely skipped.

In grade 6, students displayed numerical data in plots on a number line, including **dot plots**, **histograms**, and **box plots**. This lesson serves as a brief review of the meaning of these representations and how they are created.

In this lesson, students represent data using histograms and box plots. They compare a dot plot, box plot, and histogram that represent the same data set. They create two different histograms that represent the same data set by using different intervals in each of the histograms. Finally, students calculate values for the **five-number summary** and use those values to create box plots. When students identify the information displayed by different graphical representations, they are building knowledge about when to use appropriate tools so that they can make choices about how to represent data. In a future lesson, students will have the opportunity to create box plots and histograms using Desmos.

Students make use of structure (MP7) to connect visual representations of data sets and students reason abstractly and quantitatively (MP2) by interpreting values in the given contexts.

How will the data from Check Your Readiness, Part Two inform your delivery of this lesson?

Focus and Coherence

Building On	Building Towards
NC.6.SP.4: Display numerical data in plots on a number line. ● Use dot plots, histograms, and box plots to represent data. ● Compare the attributes of different representations of the same data.	**NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots). **NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L5 Cool-down (print 1 copy per student)

LESSON

Warm-up: Battery Life (*5 minutes*)

Instructional Routine: Notice and Wonder	
Building On: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.1

The purpose of this warm-up is to elicit the idea that the same data can be displayed in different ways, which will be useful when students create different data displays in a later activity. While students may notice and wonder many things about these images, the comparison of the three representations and interpreting the information in each representation are the important discussion points.

This prompt gives students opportunities to see and make use of structure (MP7). Specifically, they might use the structure of the three representations, particularly the structure of the horizontal number line, to find mathematically important similarities in how the same set of data is represented.

Why This Routine? The purpose of the *Notice and Wonder* routine is to make a mathematical task accessible to all students with these two approachable questions. By thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1).

Step 1

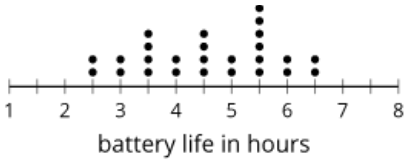
- Display the three statistical displays representing the same data set.
- Ask students to think of at least one thing they notice and at least one thing they wonder.
- Give students 1 minute of quiet think time.

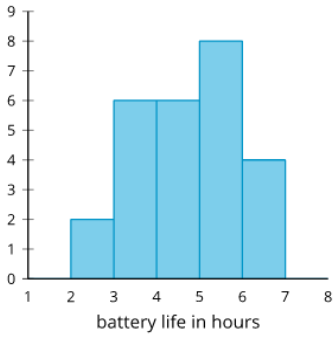
Step 2

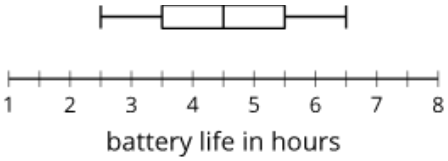
- Give students 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

The dot plot, histogram, and box plot summarize the hours of battery life for 26 cell phones constantly streaming video. What do you notice? What do you wonder?







Step 3

Facilitate a whole-class discussion.

- Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After all responses have been recorded without commentary or editing, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

The goal of this discussion is to help students recall different ways to represent distributions of data. Highlight the similarities between the dot plot and the histogram. Tell students that the tallest bar in the histogram is created from the two data values at 5 and the six data values at 5.5 in the dot plot, and that the final bar is created from the two data values at 6 and the two data values at 6.5 in the dot plot. If time permits, discuss questions such as:

- “Which representation(s) shows all the data values?” (The dot plot shows all the data values. Dot plots are useful when the data set is not too large and shows all of the individual values in the data set. In this example, a dot plot can easily show all the data. If the data set is very large (more than 100 values, for example) or if there are many different values that are not exactly the same, it may be hard to see all of the dots on a dot plot.)
- “How do you create a box plot?” (You calculate the values for the five-number summary and then graph them on a number line. The first quartile, the median, and the third quartile are used for the box, and the minimum value and maximum value are used for the whiskers.) Note: If students are not comfortable finding the five-number summary at this point in the lesson, they will have the opportunity to review the process in Activity 2.

DO THE MATH

PLANNING NOTES

Activity 1: Broad Jump Distances: Histogram (*15 minutes*)

Instructional Routine: Collect and Display (MLR2) - Responsive Strategy	
Building On: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.2

The mathematical purpose of this activity is to represent and analyze data with histograms. Students will create two different histograms from the same data set by organizing data into different intervals.

Step 1

- Ask students to form pairs or use visibly random grouping.
- Students work through the activity for 5–10 minutes.

Advancing Student Thinking: Students may struggle to know how to place numbers that lie on the boundary between intervals. For example, students may not know if a value like 160 should be included in the interval 150–160 or 160–170. Explain to students that the lower boundary value is included in the interval, and the upper boundary value is not. For example, the interval 160–170 includes all the values that are greater than or equal to 160 and less than 170.

RESPONSIVE STRATEGY

As groups work, circulate and listen to student talk about the similarities and differences between the types of data collected. Write down common or important phrases you hear students say about each type onto a visual display (e.g., "these are all numbers" or "this only has one answer"). Collect the responses into a visual display. Throughout the remainder of the lesson continue to update collected student language and remind students to borrow language from the display as needed.

Collect and Display (MLR2)

Student Task Statement

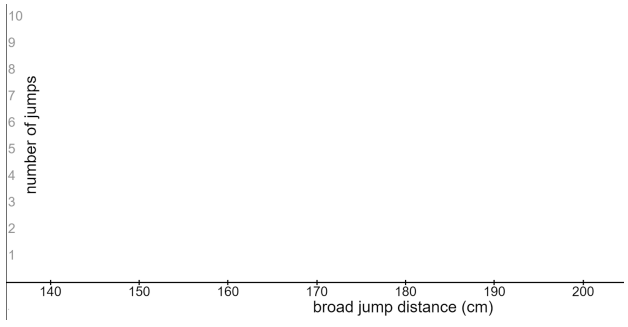
A histogram can be used to represent the distribution of numerical data.

1. The data represent the distances for all broad jumps (standing long jump), measured in centimeters (cm), at a JV track meet. Use the information to complete the frequency table.

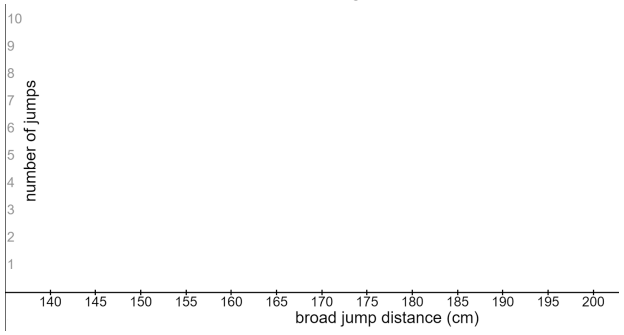
147	152	153	155	157	160	161	162
163	165	165	165	165	168	170	172
172	175	175	175	176	177	178	180
181	182	185	188	189	190		

Jump distance (cm)	Frequency
140–150	
150–160	
160–170	
170–180	
180–190	
190–200	

2. Use the set of axes and the information in your table to create a histogram.



3. The histogram you created has intervals of width 10 (like 140–150 and 150–160). Use the set of axes and data to create another histogram with an interval of width 5. How does this histogram differ from the other one?



Are You Ready For More?

It often takes some playing around with the interval widths to figure out which gives the best sense of the shape of the distribution.

1. What might be a problem with using interval widths that are too large?
2. What might be a problem with using interval widths that are too small?
3. What other considerations might go into choosing the width of an interval?

Step 2

- Facilitate a whole-class discussion. The purpose of this discussion is to make sure that students know how to create and begin to interpret histograms. Here are some questions for discussion.
  - "Where did you put the 160? In the 150–160 interval or 160–170 interval?" (Tell students that we use the convention of including the 160 in the 160–170 interval. The interval 160–170 means all the values greater than or equal to 160, but less than 170. The interval 150–160 means all values greater than or equal to 150 but less than 160.)
  - "What information is easily seen in the histogram?" (The shape of the distribution as well as estimates for the measure of center and measure of variability.)
  - "According to each histogram, what appears to be the typical broad jump length for the JV track meet?" (Using the first histogram, it appears that the typical jump length is somewhere between 160 and 180. Looking at the second histogram, it appears that the typical jump length could be between 175 and 180.)
  - "What information is not seen in the histogram?" (You are not able to see the actual values. You only know the number of values within an interval rather than the values themselves.)

DO THE MATH

PLANNING NOTES

Activity 2: Broad Jump Distances: Box Plot (*15 minutes*)

Building On: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.1

The mathematical purpose of this activity is to represent the distribution of data on the real number line with a box plot and help students think informally about the median as a measure of center. Students calculate the values for the five-number summary and create a box plot. The median, quartiles, and extreme values split the data set into four intervals with approximately the same number of data values in each. Students engage in MP2 when they interpret these values in the given context. Although these intervals are often called "quartiles," the term "quarters" is used in these materials to avoid confusion with the quartile values Q1 and Q3.

RESPONSIVE STRATEGY

To support development of organizational skills in problem solving, this task is chunked into manageable parts.

Step 1

- Keep students in pairs.
- Students work through the questions for 5 minutes. Offer students the option of working independently and comparing their answers with their partner after each question or working on the questions together.

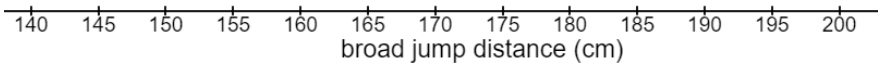
***Advancing Student Thinking*:** For students who have difficulty calculating the median, remind them that the median is the middle of a sequential data set. For students who have difficulty finding Q1 and Q3, ask them how many groups do we have if we are splitting the data into "quarters." The data should be divided into four equal groups, and the median of the lower half of the values is Q1 and the median of the upper half of the values is Q3.

Student Task Statement

A box plot can also be used to represent the distribution of numerical data.

Minimum	Q1	Median	Q3	Maximum

1. Using the same data as the previous activity for broad jump distances, find the median and add it to the table. What does the median represent for these data?
2. Find the median of the least 15 values. Because this number splits the data into the first and second quarters, it is called the first quartile. Add this value to the table under Q1. What does this value mean in this situation?
3. Find the value (the third quartile) that splits the data into the third and fourth quarters and add it to the table under Q3. Add the minimum and maximum values to the table.
4. Use the five-number summary to create a box plot that represents the length of the broad jumps in cm at the JV track meet.



Step 2

- Facilitate a whole-class discussion. The goal of the discussion is to make sure students understand the five-number summary and to help them think informally about the median as a measure of center. Here are some questions for discussion.
  - "What information is easily seen in the box plot?" (The minimum value, quartiles including the median, and the maximum value. This also highlights the interquartile range and the range.)
  - "According to the box plot, what is the typical length of a broad jump at the JV track meet?" (The typical broad jump length is 171 cm because the median of the data is 171 cm.)

 **DO THE MATH**	**PLANNING NOTES**

Lesson Debrief (*5 minutes*)

In this lesson, students viewed data represented by dot plots, histograms, and box plots. Facilitate a discussion using the following questions:

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion. • "What are the strengths of each of the representations?" (A dot plot lets you see all of the data and how they are distributed. The histogram summarizes the data into intervals that make for fewer columns. The box plot displays the five-number summary graphically.) • "What are the weaknesses of each of the representations?" (A dot plot has many columns of dots that can make it difficult to determine patterns graphically. Both the histogram and the box plot do not display each individual value in the data set which means that the mean cannot be calculated directly from either representation. • "How do you find the 'typical' value for a data set?" (You can calculate the mean or median, or estimate the mean or median using a graphical representation.) • "What is something that the data displays you've used have in common?" (Dot plots, histograms, and box plots all provide ways to look at the shape and distribution.)	**PLANNING NOTES**

Student Lesson Summary and Glossary

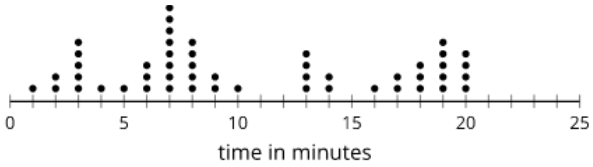
The table shows a list of the number of minutes people could intensely focus on a task before needing a break. Fifty (50) people of different ages are represented.

19	7	1	16	20	2	7	19
9	13	3	9	18	13	20	8
3	14	13	2	8	5	17	7
18	17	8	8	7	6	2	20
7	7	10	7	6	19	3	18
8	19	7	13	20	14	6	3
19	4						

In a situation like this, it is helpful to represent the **distribution** graphically to better notice any patterns or other interesting features in the data.

> **Distribution**: How many of each value or each category there are in a data set that is numerical or categorical.

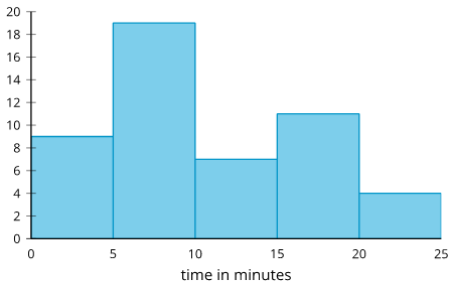
A **dot plot** shows all of the individual values in the data set:



Dot plots are useful when the data set is not too large. In this view, it is easy to see quite a few people lost focus at around 3, 7, 13, and 19 minutes, and nobody lost focus at 11, 12, or 15 minutes. If the data set is very large (more than 100 values, for example) or if there are many different values that are not exactly the same, it may be hard to see all of the dots on a dot plot.

> **Dot plot**: A way of displaying data that shows all the values in the data set. It is useful when the data set is not too large.

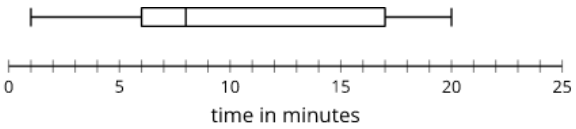
A **histogram** is useful for larger data sets and for data sets that are not restricted to whole numbers:



In this view, it is easy to see that most people lose focus between 5 and 10 minutes or between 15 and 20 minutes, while only 4 of the 50 people got distracted between 20 and 25 minutes. In a histogram, each interval includes the number at the lower end of the interval but not the upper end. For example, the tallest bar displays values that are greater than or equal to 5 minutes but less than 10 minutes.

> **Histogram**: A graphical display of data using bars of different heights. In a histogram, each bar groups numbers into intervals. In each interval the lower boundary value is included in the interval, and the upper boundary value is not.

Box plots are created using the **five-number summary**:



The vertical lines are drawn at the minimum (1 minute), Q1 (6 minutes), median (8 minutes), Q3 (17 minutes), and maximum (20 minutes). These 5 values split the data into four sections each representing approximately one-fourth of the data. Like the histogram, the box plot does not show individual data values, but other features such as quartiles, range, and median are seen more easily.

Five-number summary: These five values: the minimum value, the first quartile, the median, the third quartile, and the maximum value.

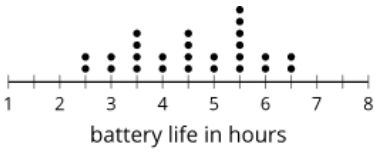
Box plot: A graphical display of data that shows the five-number summary.

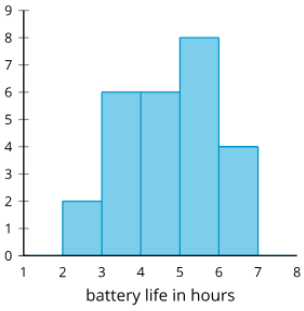
Cool-down: Reasoning About Representations (*5 minutes*)

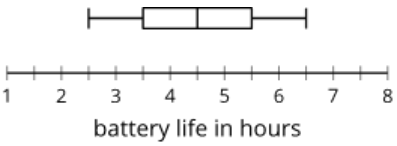
Addressing: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.1
Cool-down Guidance: More Chances Students will have more opportunities to develop language to describe the shape of a distribution and interpret data displays. Lesson 7 explicitly teaches distribution shapes, so students who are not yet describing data sets as having a shape will have lots of opportunities to explore this idea in Lesson 7.	

Cool-down

The dot plot, histogram, and box plot represent the distribution of the same data in three different ways.







1. What information can be seen most easily in the dot plot?
2. What information can be seen most easily in the histogram?
3. What information can be seen most easily in the box plot?

Student Reflection:

I felt really confident today when I was_____.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

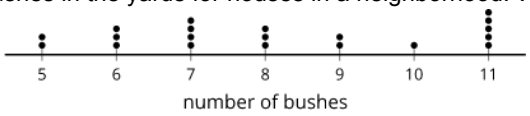
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What unfinished learning or misunderstandings do your students have about representing data? How did you leverage those misconceptions in a positive way to further the understanding of the class?

Practice Problems

1. The dot plot displays the number of bushes in the yards for houses in a neighborhood. What is the median?



2. The data set represents the shoe sizes of 19 students in a fifth-grade physical education class.
   4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7.5, 7.5, 8, 8, 8.5, 8.5, 9

   Create a box plot to represent the distribution of the data.

3. The data set represents the number of pages in the last book read by each of 20 students over the summer.
   163, 170, 171, 173, 175, 205, 220, 220, 220, 253, 267, 281, 305, 305, 305, 355, 371, 388, 402, 431

   Create a histogram to represent the distribution of the data.

4. Each set of data was collected from surveys to answer statistical questions. Select **all** of the data sets that represent numerical data.

   a. {1, 1.2, 1.4, 1.4, 1.5, 1.6, 1.8, 1.9, 2, 2, 2.1, 2.5}

   b. {Red, Red, Yellow, Yellow, Blue, Blue, Blue}

   c. {45, 60, 60, 70, 75, 80, 85, 90, 90, 100, 100, 100}

   d. {-7, -5, -3, -1, -1, -1, 0}

   e. {98.2, 98.4, 98.4, 98.6, 98.6, 98.6, 98.6, 98.7, 98.8, 98.8}

   f. {Yes, Yes, Yes, Yes, Maybe, Maybe, No, No, No}

   g. {A, A, A, B, B, B, C, C, C}

   (From Unit 1, Lesson 4)

5. Is “What is the typical distance a moped can be driven on a single tank of gas?” a statistical question? Explain your reasoning.

   (From Unit 1, Lesson 4)

6. Here is a data display representing the number of colors students are wearing.

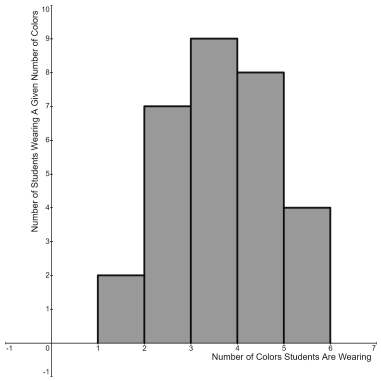
   a. How many students are wearing 1–2 colors?

   b. How many students are wearing 3–4 colors?

   c. How many students were surveyed?

   d. How might these representations look different if 200 students were surveyed?

   (Addressing NC.6.SP.4)



Lesson 6: A Gallery of Data

PREPARATION

Lesson Goal	Learning Target
• Create and interpret graphical representations of student collected data.	• I can graphically represent the data I collected and critique the representations of others.

Lesson Narrative

The mathematical purpose of the lesson is to represent and interpret data using data displays in a less-scaffolded way than in the previous lesson. The work of this lesson connects to previous work done in grade 6 where students summarized and described distributions. The work of this lesson connects to future work because students will use data displays to more formally describe the shape of distributions and to determine the appropriate measure of center and measure of variability for a given distribution. When students create and interpret a data display, they are reasoning abstractly and quantitatively (MP2) because they are creating a display and interpreting the meaning of the quantities in the display. Additionally, students make use of structure (MP7) to notice differences in distributions with the same shape, but different centers.

Which Standards for Mathematical Practice do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Building Towards
NC.6.SP.4: Display numerical data in plots on a number line. • Use dot plots, histograms, and box plots to represent data. • Compare the attributes of different representations of the same data. **NC.6.SP.5:** Summarize numerical data sets in relation to their context. **a.** Describe the collected data by: • Reporting the number of observations in dot plots and histograms. • Communicating the nature of the attribute under investigation, how it was measured, and the units of measurement	**NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots). **NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
  - Students need the numerical data collected from a statistical question in Lesson 4.
  - Tools for each student for creating a visual display that can be shared: for example, chart paper and 4 different colored markers per group of 4 students, whiteboard space and 4 different colored markers per group of 4 students, shared online drawing tool, or access to a document camera and 4 different colored pencils per group of 4 students
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L6 Cool-down (print 1 copy per student)

LESSON

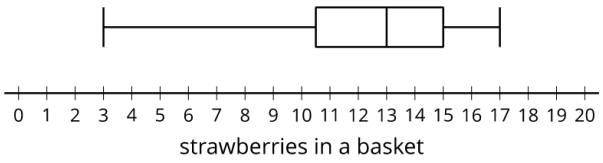
Bridge (*Optional, 5 minutes*)

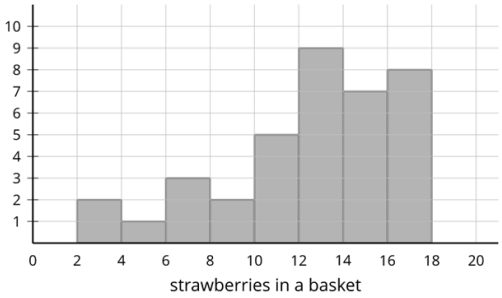
Building On: NC.6.SP.4; NC.6.SP.5a	**Building Towards:** NC.M1.S-ID.1

This bridge supports students who may need to recall the components of a histogram and a box plot, and may have struggled with questions 1–3 from Check Your Readiness. Students get to practice interpreting data by agreeing or disagreeing with statements about the information presented in data displays. This bridge connects to the cool-down from Lesson 5 in which students compared the attributes of different displays of the same data. This will be useful later in this lesson when students create their own data displays and interpret them.

Student Task Statement

There are several baskets on a table, and each basket contains a certain number of strawberries. Here are two data displays showing the number of strawberries in each basket.





Kiran makes these claims. For each claim, decide whether you agree or disagree. Explain your reasoning using at least one of the data displays.

Claims	**Agree/Disagree**	**Explain your reasoning**
a. 3 is the least amount of strawberries in any basket.		
b. The range of the number of strawberries in baskets can be found using either of the two data displays.		
c. The total number of baskets can be found using either the box plot or histogram.		

DO THE MATH

PLANNING NOTES

Warm-up: Dot Plots (*10 minutes*)

Instructional Routines: Notice and Wonder; Collect and Display (MLR2)	
Building On: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.1; NC.M1.S-ID.2

The purpose of this warm-up is to elicit the idea that distributions can be discussed in terms of shape, which will be useful when students describe data displays in a later activity. The choice of using dot plots is to ease students into describing the shape of distributions by connecting a data display they are more familiar with (dot plots) to the data displays they will be focusing on today (box plots and histograms).

While students may notice and wonder many things about these images, shape and the values on the horizontal axis are the important discussion points. This prompt gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is that data sets with different values can have distributions with the same shape if all of the values in the data set are increased or decreased by the same value.

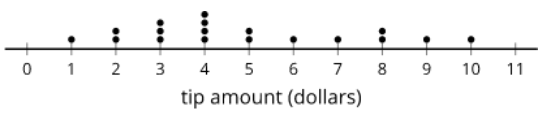
Why This Routine? The purpose of the *Notice and Wonder* routine is to make a mathematical task accessible to all students with these two approachable questions. By thinking about them and responding, students gain entry into the context and might get their curiosity piqued. Taking steps to become familiar with a context and the mathematics that might be involved is making sense of problems (MP1).

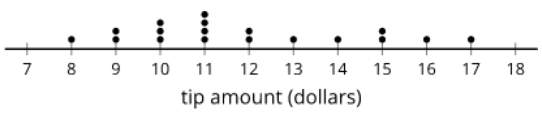
Step 1

- Display the dot plots for all to see.
- Ask students to think of at least one thing they notice and at least one thing they wonder.
- Students have 1 minute of quiet think time and then 1 minute to discuss the things they notice with their partner.

Student Task Statement

The dot plots represent the distribution of the amount of tips, in dollars, left at two different restaurants on the same night.
What do you notice? What do you wonder?





Step 2

- Facilitate a whole-class discussion.
  - Ask students to share the things they noticed and wondered.
  - Use the *Collect and Display* routine to record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information. If the shape of the distribution and the values on the horizontal axis of each dot plot do not come up during the conversation, ask students to discuss these ideas using these questions.
    - "What do you notice about the shape of each distribution?" (The data are distributed in exactly the same way in each dot plot.)
    - "What is the most frequent value in each dot plot?" ($4 and $11)
    - "What is the value of the highest tip in each dot plot?" ($10 and $17)
    - "What is the value of the lowest tip in each dot plot?" ($1 and $8)
    - "What happens if $7 is added to each of the tips in the first dot plot?" (You get the data distribution in the second dot plot.)

DO THE MATH	PLANNING NOTES

Activity 1: Data Displays (*20 minutes*)

Instructional Routine: Compare and Connect (MLR7)	
Building On: NC.6.SP.4	**Building Towards:** NC.M1.S-ID.1, NC.M1.S-ID.2

The purpose of this activity for students to understand how to represent a distribution of data using a histogram and a box plot, and to interpret each distribution in the context of the data.

This is the first time in the course that students will participate in a *Compare and Connect* routine.

COMPARE AND CONNECT	***What Is This Routine?*** The teacher facilitates a discussion about two or more pieces of student work that include distinct mathematical representations or approaches to a problem, calling attention to the correspondences among quantities, relationships, and features of the representations. Teachers should demonstrate thinking out loud (e.g., exploring why one might do or say it this way, questioning an idea, wondering how an idea compares or connects to other ideas or language), and students should be prompted to reflect and respond. ***Why This Routine?*** Use *Compare and Connect* (MLR7) to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

In this activity, students create and display a box plot and histogram using numerical data they collected from a survey question in Lesson 4. The focus of the activity is on creating graphical displays of data.

Here are sample data sets for each of the four questions from Lesson 4 if students do not have access to their data.

- On average, how many letters are in the family (last) names for students in this class? {4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 9, 10}
- On average, how many social media accounts do students in this class have? {0, 0, 1, 1, 1, 2, 2, 2, 3, 4, 4, 5, 6, 6, 8}
- About how long did it take students in this class to get to school this morning? {5, 5, 5, 5, 10, 10, 12, 15, 15, 15, 15, 25, 25, 25, 25, 30, 35, 40, 45, 55}
- On average, how many movies did each student in the class watch this summer? {0, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 4, 6, 6, 7, 7, 8, 10, 12}

Step 1

- Ask students to form small groups or use visibly random grouping. Provide each group one of the statistical questions for which students already collected numerical data.
- Provide each group with tools for creating a graphical display. Offer students options, if available.
- Each group creates a histogram and box plot to display the distribution of the data along with three comments that interpret the data. Instruct students to collaboratively create the displays and interpretations using different colors for each student in a group. The expectation is to have balanced contributions from each member of each group, demonstrated by the different colors.

RESPONSIVE STRATEGIES

Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "Each student in this class...."

Supports accessibility for: Language; Organization

Step 2

- Pause the class to discuss how to do a gallery walk and share how a group might "write a sentence or two summarizing the information in the display" to help students understand the types of responses expected. Remind students to continue to use different colors, one color per group member, when writing sentences to summarize during the gallery walk.

Step 3

- Use the *Compare and Connect* routine to ask students to consider what is the same and what is different about each display. Draw students' attention to the different ways the data are represented (box plots, histograms) and the benefit of each display for interpreting data. Focusing on connections and distinctions among the groups' displays gives students an opportunity to connect their everyday language to the language that is useful for interpreting and summarizing the data in each group's display.

***Advancing Student Thinking*:** Students may neglect including titles for axes and may forget the importance of building their plots on a number line with equally spaced intervals. Monitor for groups that do not recall the details of making the different types of displays and suggest they refer back to their work in the previous lesson.

Student Task Statement

Your teacher will assign your group a statistical question. As a group:

1. Create a histogram and box plot to display the distribution of the data.
2. Write three comments that interpret the data.

As you visit each display during the gallery walk, write a sentence or two summarizing the information in the display.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

The goals of this lesson are to make sure students understand how to represent a distribution of data using a histogram and a box plot, and to interpret each distribution in the context of the data. The purpose of the discussion is to elicit evidence of student thinking about the distributions.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "When you look at the two data displays you made, what information jumps out at you?" (The histogram shows the shape of the data. The box plot shows the median and gives me an idea of the interval that contains the middle fifty percent of the data.)
- "If you collected data from all the students in the school, instead of just your classmates, which would you rather create, a histogram or a box plot? Why?" (A box plot because all you have to do is find the five-number summary. A histogram because I do not have to order all the numbers but just decide my intervals and count how many numbers fall within each interval).
- "What are some contexts that you have seen dot plots, box plots, or histograms outside of this class?" (I have used dot plots in science class when we collected data from an experiment. I have used histograms when we were reading technical writing in English class.)
- "What do you understand about data displays?" (I can use data displays to show the distribution of data. Different displays allow me to notice information about the distribution of the data in different ways.)

PLANNING NOTES

Student Lesson Summary and Glossary

We can represent a distribution of data in several different forms, including lists, dot plots, histograms, and box plots. A list displays all of the values in a data set and can be organized in different ways.

This list shows the pH for 30 different water samples.

5.9	7.6	7.5	8.2	7.6	8.6	8.1	7.9
6.1	6.3	6.9	7.1	8.4	6.5	7.2	6.8
7.3	8.1	5.8	7.5	7.1	8.4	8.0	7.2
7.4	6.5	6.8	7.0	7.4	7.6		

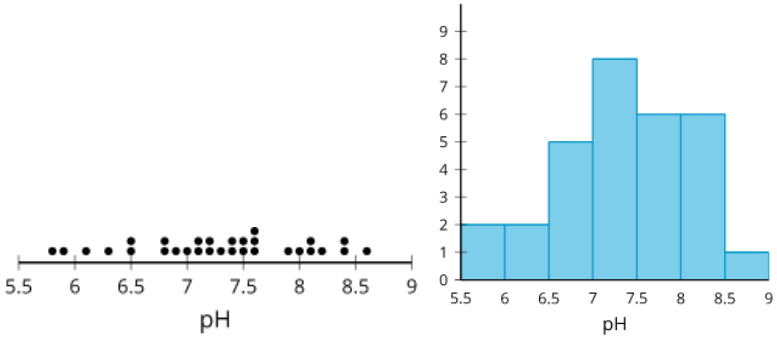
Here is the same list organized in order from least to greatest.

5.8	5.9	6.1	6.3	6.5	6.5	6.8	6.8
6.9	7.0	7.1	7.1	7.2	7.2	7.3	7.4
7.4	7.5	7.5	7.6	7.6	7.6	7.9	8.0
8.1	8.1	8.2	8.4	8.4	8.6		

With the list organized, you can more easily:

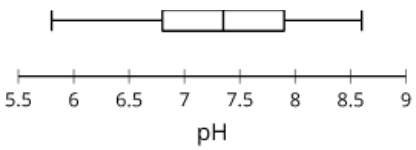
- interpret the data
- calculate the values of the five-number summary
- estimate or calculate the mean
- create a dot plot, histogram or box plot

Here is a dot plot and histogram representing the distribution of the data in the list.



To make a histogram, first decide on the intervals you will use to group data together. Count the number of values from the data set in each interval and draw a bar over that interval at a height that matches the count. In the pH histogram, there are five water samples that have a pH between 6.5 and 7 (including 6.5, but not 7).

Here is a box plot representing the distribution of the same data as the dot plot and histogram.



To create a box plot, find the five-number summary: the minimum, first quartile, median, third quartile, and maximum values for the data set. Draw a vertical mark at each of these values, then connect the pieces as in the example. For the pH box plot, we can see that the minimum is about 5.8, the median is about 7.4, and the third quartile is around 7.9.

Cool-down: Why Graphical Representations? (*5 minutes*)

Building On: NC.6.SP.4

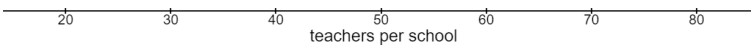
Cool-down Guidance: Points to Emphasize
Look carefully at cool-downs to ensure students are able to create histograms and box plots. Select student work to share to highlight and correct common errors at the start of the next lesson.

Cool-down

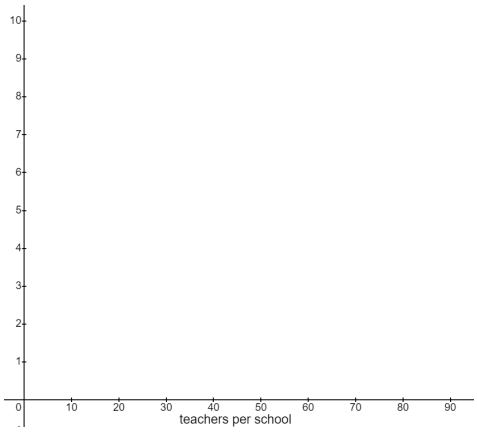
A large school system summarizes the number of teachers at 33 schools in the area.

Number of teachers	Number of schools	Five-number summary
20–29	5	minimum: 20 teachers maximum: 70 teachers median: 42 teachers Q1: 35 Q3: 60
30–39	7	
40–49	5	
50–59	6	
60–69	6	
70–79	4	

1. Create a box plot that shows this information.



2. Create a histogram that shows this information.



3. Which of these data displays most easily shows how many schools have at least 60 teachers per school? Explain your reasoning.

Student Reflection:

Pretend a classmate was absent today. What are three or four key lesson items you would share with them to help them catch up?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

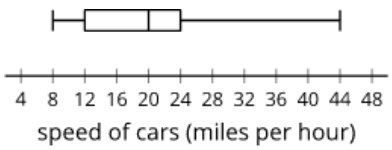
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What did you say, do, or ask during the lesson debrief that helped students be clear on the learning of the day? How did understanding the cool-down for this lesson before you started teaching today help you synthesize that learning?

Practice Problems

1. The box plot represents the distribution of speeds, in miles per hour, of 100 cars as they passed through a busy intersection.



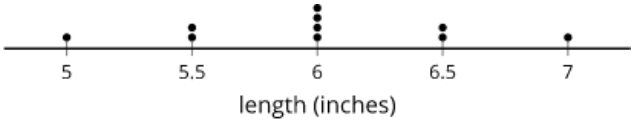
   a. What is the smallest value in the data set? Interpret this value in the situation.
   b. What is the largest value in the data set? Interpret this value in the situation.
   c. What is the median? Interpret this value in the situation.
   d. What is the first quartile (Q1)? Interpret this value in the situation.
   e. What is the third quartile (Q3)? Interpret this value in the situation.

2. The data set represents the number of eggs produced by a small group of chickens each day for ten days: 7, 7, 7, 7, 7, 8, 8, 8, 8, 9. Select **all** the values that could represent the typical number of eggs produced in a day.

   a. 7.5 eggs
   b. 7.6 eggs
   c. 7.7 eggs
   d. 8 eggs
   e. 9 eggs

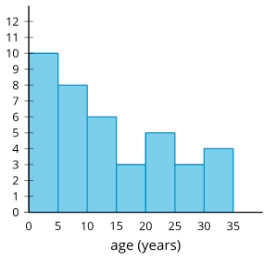
3. Choose one of the more interesting questions you or a classmate asked and collect data from a larger group, such as more students from the school. Create a data display and compare results from the data collected in class.

4. The dot plot displays the lengths of pencils (in inches) used by students in a class. What is the mean?



(From Unit 1, Lesson 5)

5. The histogram represents ages of 40 people at a store that sells children's clothes. Which interval contains the median?



   a. The interval from 0 to 5 years.
   b. The interval from 5 to 10 years.
   c. The interval from 10 to 15 years.
   d. The interval from 15 to 20 years.

(From Unit 1, Lesson 5)

6. The data set represents the responses, in degrees Fahrenheit, collected to answer the question, “How hot is the sidewalk during the school day?”

   92, 95, 95, 95, 98, 100, 100, 100, 103, 105, 105, 111, 112, 115, 115, 116, 117, 117, 118, 119, 119, 119, 119, 119, 119

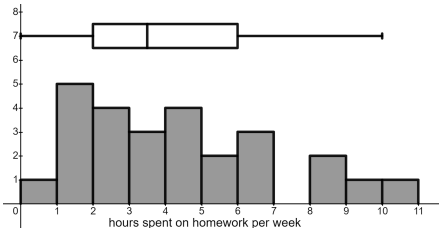
   a. Create a box plot to represent the distribution of the data.
   b. Create a histogram to represent the distribution of the data.
   c. Which display gives you a better overall understanding of the data? Explain your reasoning.

(From Unit 1, Lesson 5)

7. Is “What is the area of the floor in this classroom?” a statistical question? Explain your reasoning.

(From Unit 1, Lesson 4)

8. Here are two data displays that show the number of hours per week the same group of 26 NC Math 1 students reported spending on homework.[1]



   Kai makes these claims. For each claim, decide whether you agree or disagree. Explain your reasoning using at least one of the data displays.

Claims	Agree/Disagree	Explain your reasoning
10 is the most amount of hours spent in a week on homework.		
The range of the number of hours spent on homework per week can be found using either of the two data displays.		
The total number of students spending less than 3 hours per week on homework can be found using either the box plot or histogram.		

(Addressing NC.6.SP.4; NC.6.SP.5a)

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 7: The Shape of Distributions

PREPARATION

Lesson Goal	Learning Target
• Describe (orally and in writing) the shape of a distribution using words such as "symmetric," “skewed,” “bell-shaped,” “uniform,” and “bimodal.”	• I can describe the shape of a distribution using the terms "symmetric," “skewed,” “bell-shaped," “uniform,” and “bimodal.”

Lesson Narrative

The mathematical purpose of this lesson is to describe distributions using the appropriate terminology. In order to learn more about different kinds of distributions, one thing students do is invent reasonable contexts for a given distribution. The terminology that is used is described here. The priority in Math 1 is for students to use language for symmetric, skewed, and bell-shaped distributions.

- In a **symmetric distribution**, there is a vertical line of symmetry in the center of the data display.
- In a **skewed distribution**, one side of the distribution has more values farther from the bulk of the data than the other side.
- A **bell-shaped distribution** has a dot plot that takes the form of a bell with most of the data clustered near the center and fewer points farther from the center.
- A **uniform distribution** has the data values evenly distributed throughout the range of the data.
- A **bimodal distribution** has two very common data values seen in a dot plot or histogram as distinct peaks.

In grade 6, students may have acquired some different ways to describe distributions, though they weren’t required to learn the terminology introduced in this lesson. In a previous lesson, students created data displays. In upcoming work, students will use information about the shape of distributions to determine the appropriate measure of center. The *Which One Doesn’t Belong?* activity gives students a reason to begin using language precisely (MP6) and provides the opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. In the card sort, students trade roles explaining their thinking and listening, providing opportunities to explain their reasoning and critique the reasoning of others (MP3).

What math language will you want to support your students with in this lesson? How will you do that?

Focus and Coherence

Building On	Building Towards
NC.6.SP.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
  - Matching Distributions data display cards (print 1 copy per every 2 students and cut up in advance)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L7 Cool-down (print 1 copy per student)

LESSON

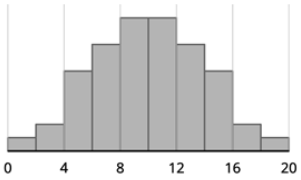
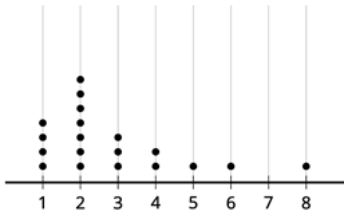
Bridge (*Optional, 5 minutes*)

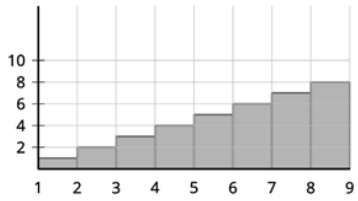
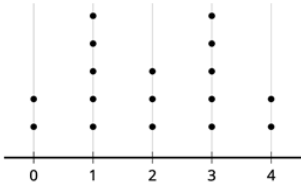
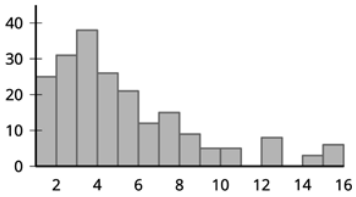
Building On: NC.6.SP.2

The purpose of this bridge is for students to use mathematical language to describe the shape of distributions. This bridge is helpful for students who may have struggled with question 1 from Check Your Readiness. This will be useful later in this lesson when students match data displays that represent the same data by identifying the shape of the distributions.

Student Task Statement

For each picture and description (a-e), state whether you agree or disagree, and explain your reasoning.

Claims	Agree/ Disagree	Explain your reasoning
a. Bell-shaped since there is a central peak for symmetric data values that are less frequent on the ends. 		
b. Symmetric because if the distribution was cut in half, both sides would be the same shape. 		

c. Uniform because there seems to be the same amount of data points across the entire distribution. 		
d. Symmetric because if the distribution was cut in half, both sides would be the same shape. 		
e. Skewed left since most of the data values are on the left side of the distribution. 		

DO THE MATH

PLANNING NOTES

Warm-up: Distribution Shape (*10 minutes*)

Instructional Routines: Which One Doesn't Belong?; Round Robin

Building Towards: NC.M1.S-ID.2

Why This Routine? *Which One Doesn't Belong?* fosters a need to define terms carefully and use words precisely (MP6) in order to compare and contrast a group of mathematical objects or representations. Because there are no wrong answers, the focus is on student reasoning, and especially on students communicating their reasoning. This routine cultivates an inclusive classroom culture by prompting students to be creative thinkers, clear communicators, and good listeners.

The mathematical purpose of this warm-up is to collect informal terminology students may use to describe shapes of distributions, as well as any ways to describe distributions they may remember from work in earlier grades. This warm-up prompts students to compare four distributions. It gives students a reason to begin using language precisely (MP6) and gives you the opportunity to hear how they use terminology and talk about characteristics of the items in comparison to one another. As a reminder, there is no correct answer to the question of which one doesn't belong. A benefit of this routine is offering an opportunity for all students to share their thinking.

Step 1

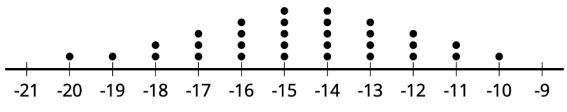
- Ask students to arrange themselves in small groups or use visibly random grouping. Display the distributions for all to see.
- Give students 1 minute of quiet think time and then 2–3 minutes to share their thinking with their small group using the *Round Robin* routine. Have each student share with the group their reasoning for why a particular item does not belong, and then together find at least one reason each item doesn't belong. Listen for students who use the statistically correct vocabulary as well as those who use informal language to describe the shapes.

Why This Routine? Engaging a group of students in collaborative problem solving, with equitable inclusion of ideas, can be challenging due to normative social status issues that place higher value on some students' contributions over others. *Round Robin* allows all students to include their rough draft ideas for solving an open-ended problem without a subset of students dominating the conversation. Knowing all ideas will be shared should motivate all students to try at least one strategy to solve a problem on their own, critical for making sense of problems and persevering in solving them (MP1). The active sharing and listening involved in this routine also provides opportunity for constructing and critiquing viable arguments (MP3).

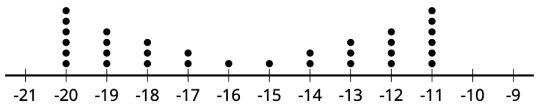
Student Task Statement

Which one doesn't belong? Explain your reasoning.

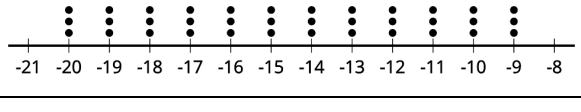
a.



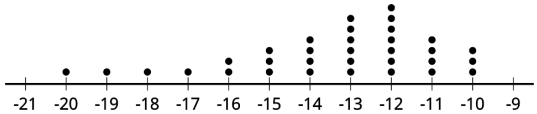
b.



c.



d.



Step 2

- Select the identified groups to share their reason why a particular item does not belong so that those with informal language speak first and those with more precise terminology follow up. Ensure that each group shares one reason why a particular item does not belong.
- Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, recast any informal language that is used to describe the shape of each distribution. Introduce and define the terms symmetric, skewed, bell-shaped, uniform, and bimodal, with an emphasis on symmetric, skewed, and bell-shaped. It is important to note that the bell-shaped distribution is also symmetric.

DO THE MATH

PLANNING NOTES

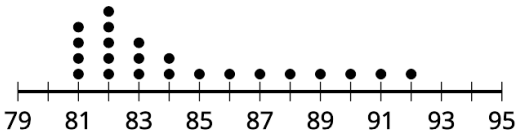
Activity 1: Matching Distributions (*20 minutes*)

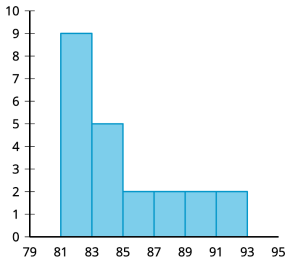
Instructional Routines: Notice and Wonder; Take Turns; Compare and Connect (MLR7)
Building Towards: NC.M1.S-ID.2

The mathematical purpose of this activity is to give students a chance to practice finding data displays that represent the distribution of the same data set and use precise vocabulary for describing the shape of the distributions while taking turns matching cards. Students trade roles explaining their thinking and listening, providing opportunities to explain their reasoning and critique the reasoning of others (MP3).

Step 1

- Ask students to form pairs or use visibly random grouping, and facilitate the *Notice and Wonder* routine.
- Display the images of the dot plot and histogram and ask them what they notice and wonder.





If it does not come up, help students notice that these two data displays show the same data in different formats. They can also be described as “skewed right” or “positively skewed.”

Step 2

- Give each pair a set of cut-up cards. Explain that a match is two different displays that represent the distribution of the same set of data.
- Students use the *Take Turns* routine: the first partner identifies a match, explains why they think it is a match, then describes the distribution while the other student listens and works to understand. Then they switch roles.

Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others’ reasoning (MP3).

Advancing Student Thinking: For students having trouble with the uniform distribution histograms, remind them that the lower bound for each interval is included and the upper bound is not. Ask them why this might change the last bar in each of these histograms. Some students may not know where to start to match data displays. You can tell them to look at the lowest and highest values as a starting point to finding similarities between two representations.

Student Task Statement

Take turns with your partner matching two different data displays that represent the distribution of the same set of data.

1. For each set that you find, explain to your partner how you know it's a match.
2. For each set that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
3. When finished with all 10 matches, write a description of the shape of each distribution.
4. Create a visual display of the sorted cards.

Step 3

- Provide time for students to investigate each others' work
- Once all pairs have completed the matching, discuss the following:
  - "What worked well when determining matches?
  - "Which matches were tricky? Explain why." (The uniform distributions may be difficult.)
  - "What vocabulary was useful to describe the shape of the distribution?" (symmetric, skewed, uniform, bimodal, bell-shaped)
  - "Were there any matches that could be described by more than one of these vocabulary terms?" (Yes, symmetric or skewed can also be used with some of the other terms for some of the distributions.)

RESPONSIVE STRATEGY

Provide students with a graphic organizer with the phrase "shapes of distributions" at the center, connecting to it all the related concepts mentioned during the discussion.

Supports accessibility for: Language; Organization

- Listen for and amplify any comments involving the use of the words symmetric, skewed, bimodal, bell-shaped, and uniform to compare the two different displays.
- Use the *Compare and Connect* routine to encourage students to make connections between the various ways of describing the different distributions. Then elicit, and revoice, the variety of language students use to compare and connect different data displays that represent the same distribution.

Why This Routine? Use *Compare and Connect* (MLR7) to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

- Display matches 2, 3, 5, and 6. Ask: Which of these distributions is most likely to show _____? Explain your reasoning.
  - the typical low temperature in a Siberian town during January
  - the number of feet below the surface where core samples were taken
  - restaurant customers' rankings of the service at a restaurant
  - the number of attempts to successfully shoot a 3 pointer in basketball on the first day of practice

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students describe the shape of distributions using formal language and reflect on some contexts for distributions with different shapes. Here are some questions for reflection and discussion.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

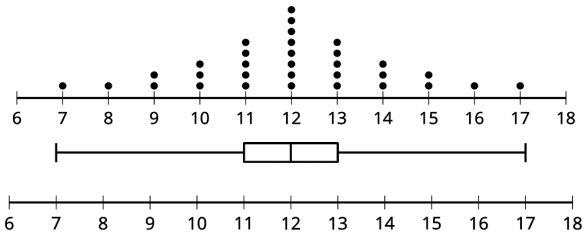
- "What does a symmetric data set look like?" (It will have a line of symmetry in the middle and the left side will look like a reflection of the right side.)
- "What does it mean to say that the shape of a distribution is uniform?" (There will be an equal number of each data value and the shape will look rectangular.)
- "Have you heard of a bell curve before? How does this relate to a bell-shaped distribution?" (Yes. I have heard of it in science class where a bell curve was used to compare data in an experiment.)
- "What is an example of a context where you would expect to find a bimodal distribution?" (You might find it if you measured the weight of a herd of cows in the springtime. The adult cows would be one peak and the calves would be the other peak.)
- "Can a skewed distribution also be symmetric? Why or why not?" (No, because skewed means that one side of the peak of the data has more data values further away from the peak than the other side. There is no line of symmetry.)

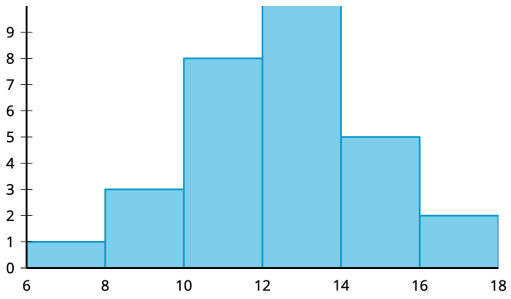
PLANNING NOTES

Student Lesson Summary and Glossary

We can describe the shape of distributions as *symmetric, bell-shaped, skewed, uniform,* or *bimodal*.

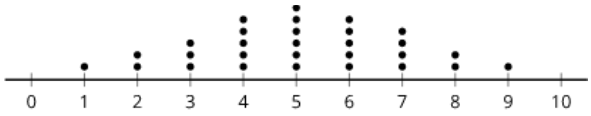
Here is a dot plot, histogram, and box plot representing the distribution of the same data set. This data set has a **symmetric distribution**, which means there is a vertical line of symmetry in the center of the data display. In the dot plot shown, the distribution is symmetric about the data value 12. The histogram does not look perfectly symmetric because data is grouped together.





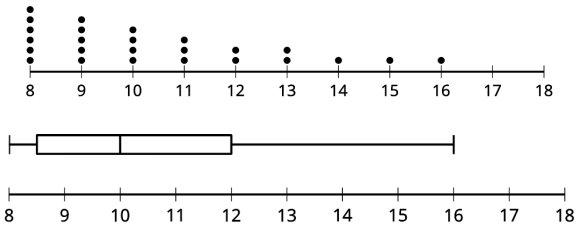
Symmetric distribution: A distribution that has a vertical line of symmetry in the center of the data display.

A **bell-shaped distribution** has a dot plot that takes the form of a bell with most of the data clustered near the center and fewer points farther from the center. Bell-shaped distributions are always symmetric or close to it.

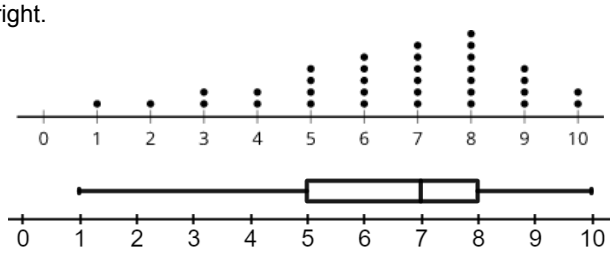


Bell-shaped distribution: A distribution that has a dot plot that takes the form of a bell with most of the data clustered near the center and fewer points farther from the center.

Here is a dot plot and box plot representing a **skewed distribution**. The data are skewed right because most of the data points are near the 8 to 10 interval, but there are many points to the right.

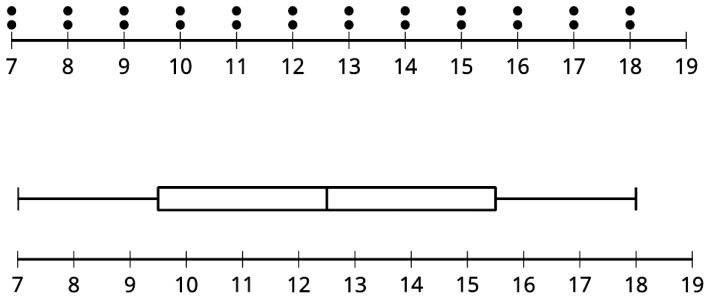


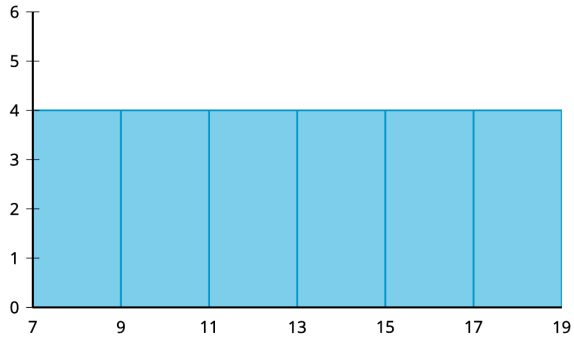
This dot plot and box plot show data that are skewed left. The data values on the left, such as 1, 2, and 3, are further from the bulk of the data than the data values on the right.



Skewed distribution: A distribution where one side of the distribution has more values farther from the bulk of the data than the other side. If the extreme values are on the right, the distribution is skewed right. If the extreme values are on the left, the distribution is skewed left.

A **uniform distribution** has the data values evenly distributed throughout the range of the data. This causes the distribution to look like a rectangle.

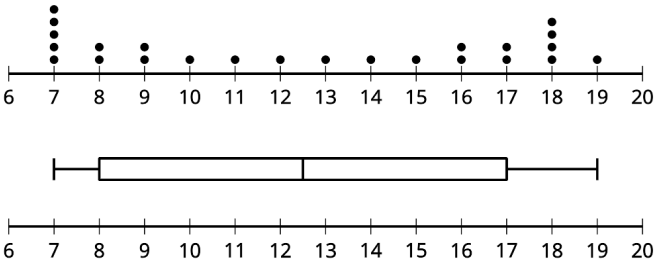


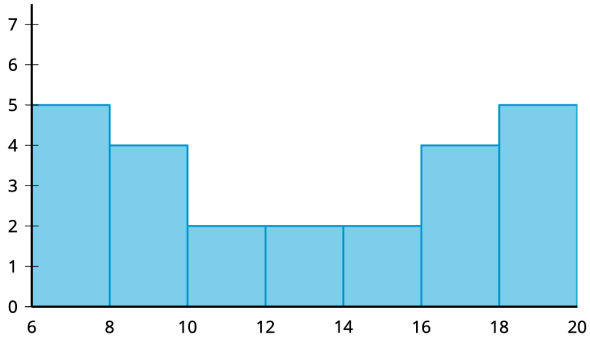


The box plot does not provide enough information to describe the shape of the distribution as uniform, though the even length of each quarter does suggest that the distribution may be approximately symmetric.

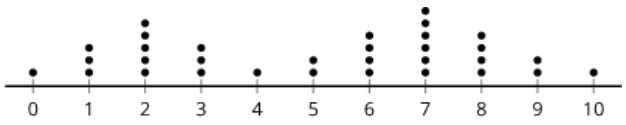
Uniform distribution: A distribution that has the values evenly distributed throughout the range of the data.

The following distribution is **bimodal**. The data cluster around two distinct peaks. Notice that you can see this in the dot plot and histogram, but not the box plot.





Bimodal distributions are not always symmetric, as in the dot plot shown below.



Bimodal distribution: A distribution that has two very common data values seen in a dot plot or histogram as distinct peaks.

Cool-down: Distribution Types (*5 minutes*)

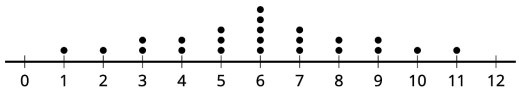
Building Towards: NC.M1.S-ID.2

Cool-down Guidance: More Chances
There will be more opportunities for students to practice this language throughout the unit. These terms may be new to students. Use visual displays and refer back to the shape of distributions, pushing for precise language. Students need to have internalized this language by Lesson 11.

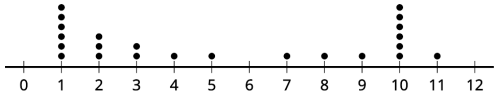
Cool-down

Describe each of these distributions. If more than one term applies, include all the terms that describe each distribution. Where possible, use these terms:

- symmetric distribution
- skewed right distribution
- skewed left distribution
- bell-shaped distribution
- uniform distribution
- bimodal distribution

1. 

2. 

3. 

4. 

5. 

6. Which of these distributions is most likely to show data collected while studying the number of plates people use while eating at an all-you-can-eat buffet? Explain your reasoning.

Student Reflection:

If you could share one thing that would help you better learn math, what would it be?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

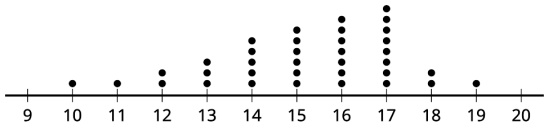
TEACHER REFLECTION

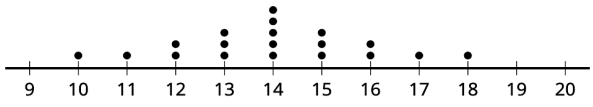
What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

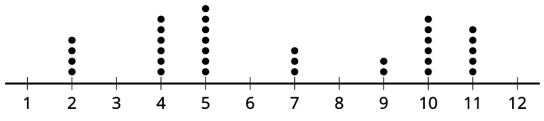
Who got to actively participate today in class and how do you know? Identify the norms or routines that allowed those students to engage in mathematics. How can you adjust these norms and routines so more students participate tomorrow?

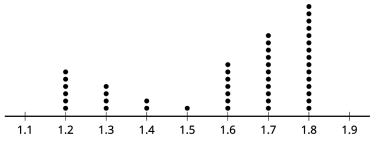
Practice Problems

1. Which of the dot plots shows a symmetric distribution?

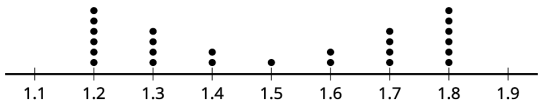
a.


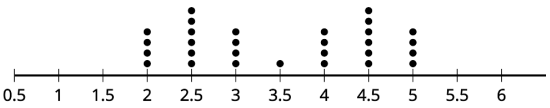
b.


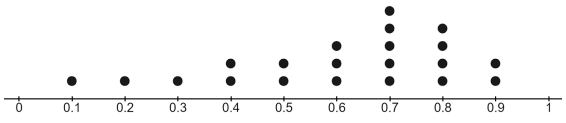
c.


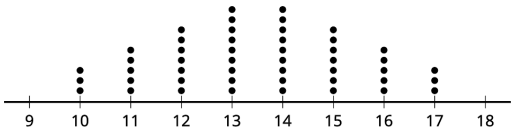
d.


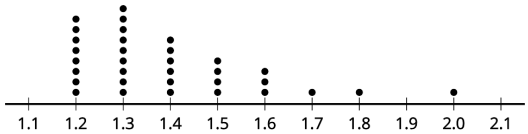
2. Which of the dot plots shows a skewed right distribution?

a.


b.


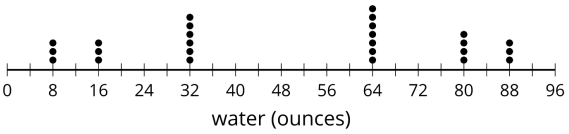
c.


d.


e.


3. The data represent the number of ounces of water that 26 students drank before donating blood: 8, 8, 8, 16, 16, 16, 32, 32, 32, 32, 32, 32, 64, 64, 64, 64, 64, 64, 64, 80, 80, 80, 80, 88, 88, 88.

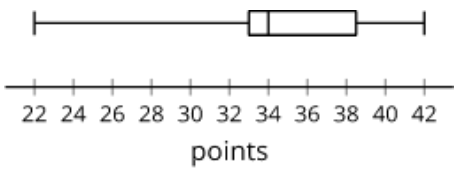
   Here is a dot plot that represents the data:



   a. Create a box plot for the data.
   b. What information about the data is provided by the box plot that is not provided by the dot plot?
   c. What information about the data is provided by the dot plot that is not provided by the box plot?
   d. It was recommended that students drink 48 or more ounces of water. How could you use a histogram to easily display the number of students who drank the recommended amount?

   (From Unit 1, Lesson 6)

4. The box plot represents the distribution of the number of points scored by a cross country team at 12 meets.



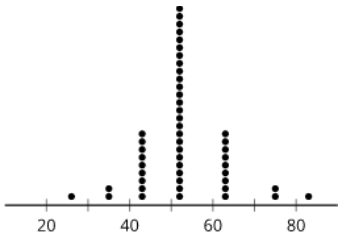
   a. If possible, find the mean. If not possible, explain why not.
   b. If possible, find the median. If not possible, explain why not.
   c. Explain why it is impossible to tell if the cross country team ever scored 30 points at a meet.

   (From Unit 1, Lesson 5)

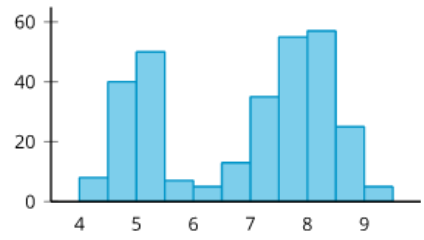
5. Describe the approximate shape of each distribution using the terms:

   - approximately
   - symmetric
   - bell-shaped
   - skewed left
   - skewed right
   - uniform
   - bimodal

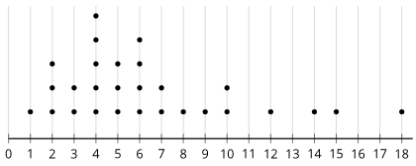
a.



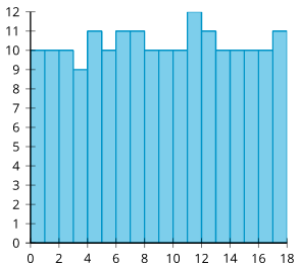
b.



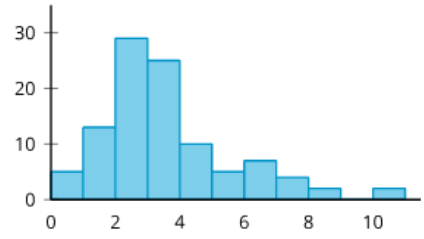
c.



d.



e.



   (Addressing NC.6.SP.2)

Lesson 8: Calculating Measures of Center and Variability

PREPARATION

Lesson Goal	Learning Target
• Calculate interquartile range, mean, and median.	• I can calculate interquartile range, mean, and median for a set of data.

Lesson Narrative

This lesson revisits prior-grade-level content. If the Check Your Readiness diagnostic questions 5 and 6 indicate that your students know these representations well, this lesson may be safely skipped.

This lesson reinforces for students how to calculate the **mean**, **median**, and **interquartile range (IQR)**. This lesson connects to upcoming work because students will interpret data using measures of center and measures of variability throughout the unit, so it is important that they understand what they are interpreting.

In what ways might this lesson give students opportunities to surprise you with their thinking or reasoning?

Focus and Coherence

Building On	Building Towards
NC.6.SP.3: Understand that both a measure of center and a description of variability should be considered when describing a numerical data set. **a.** Determine the measure of center of a data set and understand that it is a single number that summarizes all the values of that data set. • Understand that a mean is a measure of center that represents a balance point or fair share of a data set and can be influenced by the presence of extreme values within the data set. • Understand the median as a measure of center that is the numerical middle of an ordered data set. **NC.6.SP.5:** Summarize numerical data sets in relation to their context. **b.** Analyze center and variability by: • Giving quantitative measures of center, describing variability, and any overall pattern, and noting any striking deviations. • Justifying the appropriate choice of measures of center using the shape of the data distribution. *(continued)*	**NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots). **NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

NC.7.SP.3: Recognize the role of variability when comparing two populations. **a.** Calculate the measure of variability of a data set and understand that it describes how the values of the data set vary with a single number. ● Understand the mean absolute deviation of a data set is a measure of variability that describes the average distance that points within a data set are from the mean of the data set. ● Understand that the range describes the spread of the entire data set. ● Understand that the interquartile range describes the spread of the middle 50% of the data.	

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*10 minutes*)
  - *Responsive Strategy*: Heartbeats graphic organizer (print as needed)
- **Activity 2** (*10 minutes*)
  - 1 meter stick and 14 pennies taped to stick (or other small weights)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L8 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.6.SP.3.a; NC.7.SP.3.a

This bridge supports students who may need to recall the steps to calculate mean, interquartile range, and median, which is useful for students who may have struggled with question 2 from Check Your Readiness. This bridge will be useful later in this lesson when students use given data to calculate interquartile range and median.

Student Task Statement

The first column of the table below is a list of measures of center and measures of variability. The second column describes the steps you take to calculate the measures. Match each measure with the way it is computed.

Measures	Calculations	Match
1. Median 2. Interquartile range 3. Mean	a. Add up all of the values in a data set, then divide by the number of values in the set. b. The difference between the first and the third quartiles. c. List the values in the data set in order, then find the middle value. If there are two "middle values," find the mean of those two values.	1. _____ 2. _____ 3. _____

DO THE MATH

PLANNING NOTES

Warm-up: Calculating Centers (*10 minutes*)

Instructional Routine: Aspects of Mathematical Modeling	
Building On: NC.6.SP.5.b	**Building Towards:** NC.M1.S-ID.1; NC.M1.S-ID.2

The goal of this warm-up is to review how to calculate mean and median as well as identify common mistakes in the calculation of mean and median.

This is the first time in the course that students will participate in an *Aspects of Mathematical Modeling* routine.

ASPECTS OF MATHEMATICAL MODELING

What Is This Routine? In activities tagged with this routine, students engage in scaled-back modeling scenarios, for which students only need to engage in a part of a full modeling cycle. For example, they may be selecting quantities of interest in a situation or choosing a model from a list.

Why This Routine? Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.

Step 1

- Display one problem at a time. Give students approximately 30 seconds of quiet think time for each problem.
- Discuss each statement with the whole class before moving on to the next statement.

Advancing Student Thinking: Some students may forget to sort the data when finding the median. Ask them, “What is a median? What does it tell you about the data?” Some students may not remember how to find the median when there is an even number of data values. Ask them, “What does the median tell you about the data? How could we find a middle number between these two values?”

Student Task Statement

Decide if each statement is true or false. Explain your reasoning.

1. The mean can be found by adding all the numbers in a data set and dividing by the number of values in the data set.
2. The mean of the data in the dot plot is 4.

3. The median of the data set 4, 5, 9, 1, 10 is 9.
4. The median of the data in the dot plot is 3.5.

Step 2

- Facilitate whole-class discussion for students to recall what information the mean and median reveal about the data.
  - "What does the mean tell you about the data?" (On average, where the center of the data is.)
  - "What does the median tell you about the data?" (Half of the numbers are greater than or equal to the median and half are less than or equal to the median.)
- Share some common mistakes to avoid:
  - Not putting the numbers in order when finding the median.
  - If two numbers are in the middle, forgetting to add, then divide by two to find the median.
  - Finding the middle number on the horizontal axis rather than in the data.
  - Rounding the mean to the nearest whole number.

DO THE MATH	PLANNING NOTES

Activity 1: Heartbeats (*10 minutes*)

Building On: NC.6.SP.5.b, NC.7.SP.3.a	**Building Towards:** NC.M1.S-ID.2

The purpose of this activity is to get students to calculate the median and IQR.

The data sets in this lesson are small enough that finding summary statistics like measures of center or measures of variability are not necessary. The entire data set could be assessed fairly easily and does not need further analysis. The sets are small here for the purposes of practicing calculating and understanding the statistics. In reality, finding such statistics are much more useful when the data set is much larger. For example, some devices will find heart rate every 10 minutes, giving more than 1,000 values for a week that might be analyzed to determine health information about a person. It would be difficult to understand all of the data by looking at a table or even a data display. Having some summary statistics such as mean or median would be useful to understand a person's heart rate for the week.

RESPONSIVE STRATEGY

Provide students with the graphic organizer for students to organize their work with calculating the median, quartiles, and IQR.

Supports accessibility for: Language; Organization

In this activity, an optional graphic organizer is provided as a blackline master to help students compute the interquartile range. The boxes that are shaded are not to be used and the position of the open boxes are meant to highlight the useful data for calculating the value for the row. For example, to compute the first quartiles, students use the average of the second and third values, so those boxes are left open to indicate that these data are useful.

Step 1

- Ask students to share what they know about heart rates. If time allows, have students calculate their heart rate by having them count beats for 30 seconds and then doubling their count.
- Provide students 3–5 minutes of quiet time to work the first question and then pause for a brief whole-class discussion about how to calculate the median, quartiles, and IQR.

Advancing Student Thinking: Students may have difficulty calculating the median of a data set with an even number of data points. Ask them what the median represents for the data set and where that value might be. If they still struggle, remind them that the median is the average of the two middle numbers.

Student Task Statement

The heart rates of eight high school students are listed in beats per minute:

72 75 81 76 76 77 79 78

1. What is the interquartile range?
2. How many values in the data set are:
   a. less than Q1?
   b. between Q1 and the median?
   c. between the median and Q3?
   d. greater than Q3?

Step 2

- Facilitate a whole-class discussion on the method of calculating median and IQR as well as the interpretation of each. Here are some questions for discussion.
  - "What does the IQR tell you about a data set?" (The IQR is a measure of variability which describes the range of the middle half of the data. In this case, heartbeats in the middle half of the data vary by 3 beats per minute.).
  - "How do the median and quartiles divide the data?" (Into equal quarters so that the same number of values are in each quarter.)
  - "How much of the data set is between Q1 and Q3?" (The middle 50% of the data is between these values.)

Are You Ready For More?

1. A pod of dolphins contains 800 dolphins of various ages and lengths. The median length of dolphins in this pod is 5.8 feet. What information does this tell you about the length of dolphins in this pod?
2. The same vocabulary test with 50 questions is given to 600 students from fifth to tenth grades, and the number of correct responses is collected for each student in this group. The interquartile range is 40 correct responses. What information does this tell you about the number of correct responses for students taking this test? What would a box plot look like for the results of the vocabulary test?

DO THE MATH

PLANNING NOTES

Activity 2: Pennies on a Stick (*10 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1)	
Building On: NC.6.SP.5.b, NC.7.SP.3	**Building Towards:** NC.M1.S-ID.2

The purpose of this activity is for students to visualize and interpret the mean of a data set. Students explore a strategy to estimate the mean of a distribution especially in cases where the specific data values are unknown. To estimate the mean of a distribution, especially when given a histogram, is to think about a balance point. This would be where someone might place their hand to balance the histogram. In this activity, the pennies are representative of the histogram and balancing the meter stick models how to estimate the mean.

This is the first time in the course that students will participate in a *Stronger and Clearer Each Time* routine.

STRONGER AND CLEARER EACH TIME 	***What Is This Routine?*** Students write a first draft response to a prompt, then engage in successive pair-shares to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original response. Throughout this process, students should be encouraged to press each other for clarity and details. ***Why This Routine?*** *Stronger and Clearer Each Time* (MLR1) provides a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. The routine provides a purpose for student conversation and fortifies student output.

Step 1

- Students work on the two questions for 3–4 minutes. Give students the choice of working independently or with a partner.

Student Task Statement

Suppose there are six pennies taped onto a meter stick so that the mean position is the 50-centimeter mark. What information does this tell you about the position of the pennies along the meter stick?

1. Find possible locations for the six pennies.
2. Find a different set of possible locations for the six pennies.

Step 2

- Display a prepared meter stick so that the centers of the pennies are taped onto the stick at {25, 25, 25, 35, 40, 40, 40, 40, 45, 65, 75, 75, 80, 90}. Show how the stick balances when you put your finger at the 50 centimeter mark and how some pennies are closer to or farther from the mean.
- Create a display that incorporates the measures of center (mean and median) and variability (interquartile range). Later in Lesson 12, standard deviation will be added as a measure of variability. This display should be posted in the classroom for the remaining lessons within this unit. (See displays in the Student Workbook Lesson 12 Summary and Glossary as an example.)

Step 3

- Use the *Stronger and Clearer Each Time* routine to give students a structured opportunity to revise and refine their response to the question, “If you put pennies at 45, 35, and 70, explain how you know the mean is different from the median.”
- Ready students to meet in pairs with two or three groups of partners in a row, for a series of brief, 1–2 minute conversations about their initial responses to the prompt.
  - For each round, partners ask each other clarifying questions and provide additional ideas.
  - Encourage students to take notes for themselves on feedback and useful ideas from each conversation.

- Display suggested prompts for feedback that will help students strengthen their ideas and clarify their language:
  - When you said _____, what did you mean?
  - I was confused when you said _____ because _____ .
  - Can you give an example of what you mean?
  - What else do you think (or know) is true?
- After receiving feedback from two or three partners, ask each student to write a second draft response that is stronger and clearer than their initial response. Encourage students to borrow ideas and language from their partners as they write their own improved drafts.
- Additional questions that could be discussed.
  - "If you put pennies at 45, 35, and 70, where do you need to put a penny for the meter stick to balance at 50 cm?" (50 cm)
  - "If you put two pennies at 60, where do you need to put a penny to make the meter stick balance at 50 cm?" (30 cm)

DO THE MATH	PLANNING NOTES

Lesson Debrief (*5 minutes*)

The goals of this debrief are to make sure students know how to calculate IQR and that it is a measure of variability.

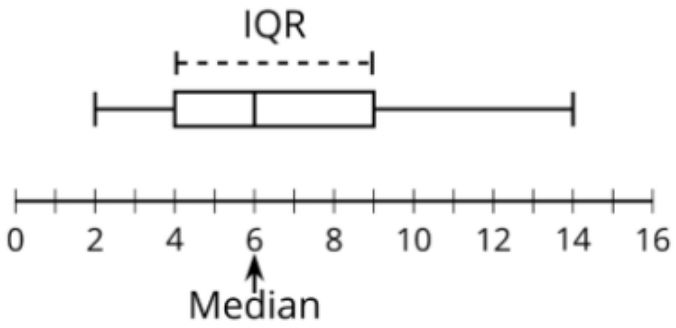
	PLANNING NOTES
Facilitate a discussion using the following questions. Decide whether students should first have an opportunity to reflect on these questions in their workbooks or talk through them with a partner. Choose what questions will be prioritized in the full class discussion. • "How do you calculate the IQR?" (Find the difference between the value of the third and first quartiles.) • "One data set has a greater IQR than another. What does this mean about the data in the first data set?" (It has greater variability. This means that the middle half of the data is more spread out from the center than the middle half of the data from the second data set.) • "How do you calculate the median when there are an even number of values in the data set?" (find the middle two numbers, then take their average) • "Which of the measures we talked about today can represent a balance point?" (the mean, we put pennies on a meter stick and it balanced at the mean position of the pennies)	

Student Lesson Summary and Glossary

In this lesson, we focused on the **interquartile range**, or **IQR**.

To calculate the IQR, subtract the value of the first quartile from the value of the third quartile. Recall that the first and third quartile are included in the five number summary.

The IQR is a measure of variability. Measures of variability tell you how much the values in a data set tend to differ from one another. A greater measure of variability means that the data are more spread out while a smaller measure of variability means that the data are more consistent and close to the measure of center.

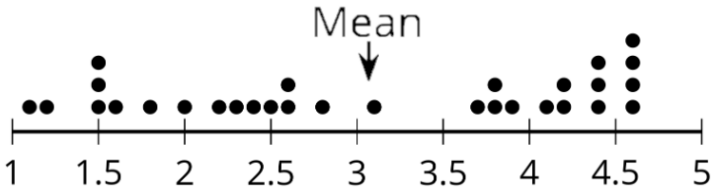


Interquartile Range (IQR): A measure of variability calculated by subtracting the first quartile from the third quartile.

Also in this lesson, we revisited two terms from previous learning:

Median: A measure of center that divides the data so that half of the values are greater than the median and half of the values are less than the median. Medians are easiest to see in a box plot.

Mean: Also called the average, it is the value you get by adding up all the values in the set and dividing by the number of values in the set. The mean also represents the balance point of a distribution.



Cool-down: Calculating Mean, Median, and IQR (*5 minutes*)

Building On: NC.6.SP.5.b, NC.7.SP.3.a	**Building Towards:** NC.M1.S-ID.2
Cool-down Guidance: Press Pause Students will have more opportunities with IQR and the concept of variability; however, students who are unable to calculate and interpret the mean and median would benefit from teacher-led small-group instruction during the next lesson.	

Cool-down

Mai ran a 2-mile trail once a week in preparation for an upcoming race. Here are Mai's times rounded to the nearest minute.

11 24 22 15 14

1. Find the median of the data. Use words to explain what that describes in terms of Mai's running times.
2. Find the mean of the data. Use words to explain what that means in terms of Mai's running times.
3. Find the interquartile range.

Student Reflection:

After today's lesson I feel: _____.

a. Very strong. b. I am getting better and will continue to do so. c. I am struggling to make progress.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

In upcoming lessons, students will find the Standard Deviation for a data set. What understandings are necessary for students to have in order to be successful at that?

Practice Problems

1. The data set represents the number of errors on a typing test.

   5 6 8 8 9 9 10 10 10 12

   a. What is the median? Interpret this value in the situation.

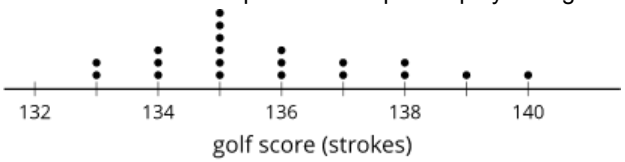
   b. What is the IQR?

2. The data set represents the heights, in centimeters, of ten model bridges made for an engineering competition.

   13 14 14 16 16 16 16 18 18 19

   What is the mean?

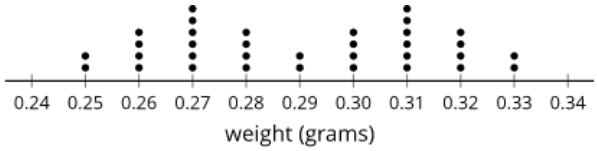
3. Describe the shape of the distribution shown in the dot plot. The dot plot displays the golf scores from a golf tournament.



   (From Unit 1, Lesson 7)

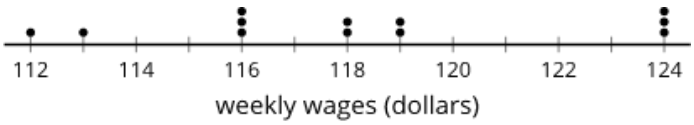
4. The dot plot shows the weight, in grams, of several different rocks. Select **all** the terms that describe the shape of the distribution.

   a. bell-shaped
   b. bimodal
   c. skewed
   d. symmetric
   e. uniform



   (From Unit 1, Lesson 7)

5. The dot plot represents the distribution of wages earned during a one-week period by 12 college students.



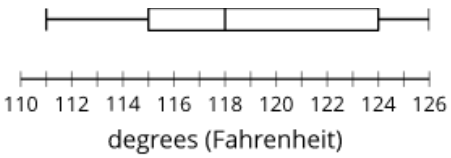
   a. What is the mean? Interpret this value based on the situation.

   b. What is the median? Interpret this value based on the situation.

   c. Would a box plot of the same data have allowed you to find both the mean and the median?

   (From Unit 1, Lesson 6)

6. The box plot displays the temperature of saunas in degrees Fahrenheit. What is the median?



   (From Unit 1, Lesson 5)

7. Here are the shoe sizes of some grade 9 and grade 12 students.

Grade 9 shoe sizes:
6, 8, 6.5, 7.5, 7, 6.5, 9, 6, 8.5, 7.5, 8, 10, 11, 8, 9

Grade 12 shoe sizes:
10, 9, 10.5, 8.5, 10, 9, 9.5, 8, 8, 11, 9, 9.5, 11, 10.5, 8.5

a. Create a box plot or histogram to represent both sets of data.

b. Complete the table.

	Mean	**Median**	**IQR**
Grade 9 shoe sizes			
Grade 12 shoe sizes			

c. Does one grade's shoe sizes have more variation than the other? Explain how you know.

(Addressing NC.6.SP.3.a; NC.7.SP.3.a)

Lessons 9 & 10: Checkpoint

PREPARATION

Lesson Goal	Learning Targets
• Use technology tools to graphically represent data and calculate useful statistics.	• I can create graphic representations of data and calculate statistics using technology. • I can apply what I have learned to analyze and create graphical representations of data sets to answer questions and make informed decisions.

Lesson Narrative

This is a Checkpoint day. Checkpoint days consist of two lessons (one full block) and are structured as four 20-minute stations that students rotate between. There are a total of seven stations students can engage with. Since students will not be able to participate in all seven stations, please note that Station A (Unit 2 Check Your Readiness) and Station B (Technology) are both required for all students.

A. Unit 2 Check Your Readiness (*Required*)
B. Technology (*Required*)
C. Teacher-led Small-group Instruction
D. Where Did the Distribution Come From?
E. Who Am I?
F. Are You Ready For More?
G. Long Jump

Where do you see connections to this lesson from what students shared and discussed in previous lessons?

Agenda, Materials, and Preparation

- **Station A** (*Required, 20 minutes*)
  - Unit 2 Check Your Readiness (print 1 copy per student)
- **Station B** (*Required, 20 minutes*)
  - Statistical technology is required for this station: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device. The technology will be used for calculating measures of center (mean and median), measures of variability (standard deviation and IQR), and creating data displays (histograms and box plots).
- **Station C** (*20 minutes*)
  - Formative student data collected from Lessons 2–8
- **Station D** (*20 minutes*)
  - Matching Distributions data display cards from Lesson 7 (print 1 copy per every 2 students and cut up in advance)
  - Matching Distributions survey question cards (print 1 copy per every 2 students and cut up in advance)
- **Station E** (*20 minutes*)
  - "Who I Am" (print 1 copy per student)
  - Colored pencils, crayons, highlighters
- **Station F** (*20 minutes*)
  - Are You Ready For More? tasks in Student Workbook from past lessons (or optional: print 1 blackline master per student)
- **Station G** (*20 minutes*)

STATIONS

Station A: Unit 2 Check Your Readiness (*Required, 20 minutes*)

Remind students that it is really important that their responses to these questions accurately represent what they know. Ask them to answer what they can to the best of their ability. If they get stuck, they should name what they don't know or understand.

Station B: Technology (*Required, 20 minutes)*

Addressing: NC.M1.S-ID.1; NC.M1.S-ID.2	**Building Towards:** NC.M1.S-ID.3

The mathematical purpose of this station is to use technology to calculate statistics, create data displays, and to investigate how those change when values are added or removed from the data set. Students will discuss or reflect on the relationship between outliers and the measure of center. Students are likely familiar with the Desmos graphing calculator (www.desmos.com/calculator), however, creating box plots and histograms using this technology may be new.

- Students choose to either work individually or with a partner.
- Provide students with instructions for accessing Desmos online.

***Advancing Student Thinking*:** Students may need guidance with formatting in Desmos the first time they use it.

Station B

Access www.desmos.com/calculator on an internet browser.

To create a list in Desmos, first name the list and set it equal to the set of numbers. For example: a data set of the numbers 4, 3, 2, 4, 1 would be entered as L=[4, 3, 2, 4, 1]. You may use any letter you'd like to name your list but you must include the equal sign and the square brackets with numbers separated by commas.

1. Create a list of data using the numbers from the set of numbers below.

   Data Set: [7, 8, 4, 13, 5, 15, 14, 8, 12, 2, 8, 13, 12, 13, 6, 1, 9, 4, 9, 15, 21, 7, 9, 2, 5, 1, 9, 8, 9, 11, 4, 10, 6, 3, 13, 9, 12, 4, 6, 8, 7, 11]

2. On a new line, type “boxplot(L)” to create a box plot of the data set. Remember, if you named your data set with a different letter, that letter should be in parentheses instead of L.
   - How many values are in the data set? Explain your reasoning.
   - On a new line, type “mean(L)”
     - What is the mean of the data set?
   - On a new line, type “stats(L)”
     - What is the minimum of the data set?
     - What is the first quartile of the data set?
     - What is the median of the data set?
     - What is the third quartile of the data set?
     - What is the maximum of the data set?
   - Use the Q1 and Q3 values to calculate the interquartile range (IQR). What is the IQR?
   - Next, add the number 32 to your data set.
     - What happens to the mean?
     - What happens to the median?
     - Discuss with your partner or reflect individually: Which center of measure (mean or median) was most affected by the addition of the 32? Why?

3. Open up www.desmos.com/calculator on a new tab.
   - From the new blank desmos screen, create a data set using the same set of numbers, without the number 32.
   - Create a histogram of the data by typing “histogram(L)” on a new line.
   - Change the Bin Alignment to Left.
   - Describe the shape of the data.
   - Click on Bin Width. Notice that the notation changes to histogram(L,1). Replace 1 with 2 so it reads histogram(L,2).
   - How did the display change as a result of adjusting the Bin Width?
   - Discuss with your partner or reflect individually: Why might changing the Bin Width be useful when representing a data set?

4. Open up www.desmos.com/calculator on a new tab to answer the following questions about the data set below.

   The data below represent the amount of corn, in bushels per acre, harvested from different locations.

   133, 133, 134, 134, 134, 135, 135, 135, 135, 135, 135, 136, 136, 136, 137, 137, 138, 138, 139, 140
   - Use technology to create a histogram and a box plot.
   - What is the shape of the distribution?
   - Compare the information displayed by the histogram and box plot.
   - What is the median number of bushels of corn harvested?
   - What is the interquartile range?
   - Discuss with your partner or reflect individually: Did you find the median and IQR using the histogram or box-plot? Why did you use that display?

5. If you chose to work independently, spend the last 5 minutes of the station talking with another student or pair to review your thinking to the questions above.

Station C: Teacher-led Small-group Instruction (*20 minutes*)

Use student cool-down data and informal formative assessment data from Lessons 2–8 to provide targeted small-group instruction to students who demonstrate the need for further support on topics taught up to this point.

Potential topics:

- Calculating and interpreting mean, median, and IQR
- Describing the shape of data distributions
- Creating box plot using the five-number summary
- Comparing and interpreting dot plots, histograms, and box plots

Station D: Where Did The Distribution Come From? (*20 minutes*)

Instructional Routine: Card Sort
Building Towards: NC.M1.S-ID.2

The mathematical purpose of this station is to remind students of the importance of context to statistics. Although some analysis can be done outside of a context, it is often useful to think about the real situations in which the data was collected to engage student intuition and understanding. The survey card questions are the actual survey questions that produced each distribution from the matched sets in Lesson 7.

This is the first time in the course that students will participate in a *Card Sort* routine.

RESPONSIVE STRATEGY

To help get students started, display sentence frames such as, "A possible survey question that produced this data is ____ because ____."

Supports accessibility for:
Language; Organization

CARD SORT

What Is This Routine? A *Card Sort* uses cards or slips of paper that can be manipulated and moved around (or the same functionality enacted with a computer interface). It can be done individually or in small groups. Students put things into categories or groups based on shared characteristics or connections. This routine can be combined with *Take Turns*, such that each time a student sorts a card into a category or makes a match, they are expected to explain the rationale while the group listens for understanding. The first few times students engage in these activities, the teacher should demonstrate how the activity is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.

Why This Routine? A *Card Sort* provides opportunities to attend to mathematical connections using representations that are already created, instead of expending time and effort generating representations. It gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

Station D

With a partner:

1. Choose a survey card. This is a question that resulted in one of the matched sets of data displays.
2. Discuss the card and make an educated guess about which set of data displays represents the data from that survey question.
3. Explain your choice. Use the questions below as a guide for your explanation:
    - How did you use the shape of the data to come up with your question?

- Would you always expect your question to result in a [symmetric, skewed, bell-shaped, bimodal, uniform] distribution?

4. There is one survey card for each set of data displays.

5. Repeat the process, as time permits.

Survey question	Data display	Explanation
a.		
b.		
c.		
d.		

DO THE MATH

PLANNING NOTES

Station E: Who I Am?[1] (*20 minutes*)

- The purpose of this station is to further build community in the classroom. Therefore, if this station is offered to students, ensure that every student in the class has the opportunity to participate.
- In this station, students complete the "Who I Am" template. Encourage students to use color/patterns, draw pictures, and really make it their own.
- After this lesson, display these around the classroom so students can learn more about one another. Ensure all students are able to complete their collage. If needed, students could work on it outside of class.
- Have students reference back to these templates for the Personality Coordinates Icebreaker in the Post-test Activities lesson (Lesson 16) of this unit.

[1] Adapted from Dan Meyer

Station F: Are You Ready For More? (*20 minutes*)

Students who did not complete the "Are You Ready For More?" task statements from Lessons 5 and 8 can do so in Station F. This is a great opportunity for students to expand their thinking. These tasks can also be offered as additional practice problems or used in the teacher-led small-group instruction.

Station F

1. When creating a histogram, it often takes some playing around with the interval lengths to figure out which gives the best sense of the shape of the distribution.
   a. What might be a problem with using interval lengths that are too large?
   b. What might be a problem with using interval lengths that are too small?
   c. What other considerations might go into choosing the length of an interval?

   (From Unit 1, Lesson 5)
2. A pod of dolphins contains 800 dolphins of various ages and lengths. The median length of dolphins in this pod is 5.8 feet. What information does this tell you about the length of dolphins in this pod?

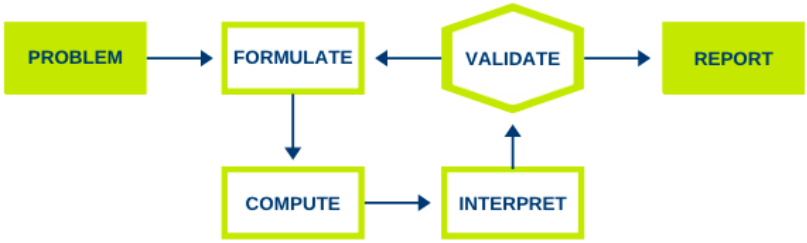
   (From Unit 1, Lesson 8)
3. A vocabulary test with 50 questions is given to 600 students from fifth to tenth grades and the number of correct responses is collected for each student in this group. The interquartile range is 40 correct responses. What information does this tell you about the number of correct responses for students taking this test? What would a box plot look like for the results of the vocabulary test?

   (From Unit 1, Lesson 8)

Station G: Long Jump[2] (*20 minutes*)

Instructional Routine: Aspects of Mathematical Modeling

This task moves students quickly between the formulate, compute, and interpret stages of the modeling cycle. It serves as an example that these stages are not always performed in isolation or in a predicted sequence. Modeling is the link between the mathematics students learn in school and the problems they will face in college, career, and life. Time spent on modeling in Math 1 is crucial as it prepares students to use math to handle technical subjects in their further studies, and problem solve and make decisions that adults need to make regularly in their lives.



This task can also be offered as an additional practice problem at any point in the unit or used in the teacher-led small group instruction.

Why This Routine? Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.

[2] Adapted from AchievetheCore.org

Station G

The track and field coach has to select a girl for the long jump at the regional championship. Three girls are in contention. The results of a school jump-off, in meters, are given in the accompanying table.

Elena	Jada	Priya
3.25	3.55	3.67
3.95	3.88	3.78
4.28	3.61	3.92
2.95	3.97	3.62
3.66	3.75	3.85
3.81	3.59	3.73

1. Who do you think should go to the championship? Write a letter to the coach with your decision and an explanation of your reasoning.
2. What are the long-jump records for your school, district, and/or state? When did they occur? Be sure to include the units. What do you notice and wonder about what you discovered?

DO THE MATH

PLANNING NOTES

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

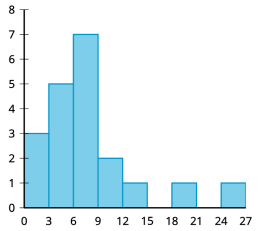
When do your students feel successful in math? How do you know?

Practice Problems

1. *Technology required.* The data represent the average customer ratings for several items sold online.

0.5	1	1.2	1.3	2.1	2.1	2.1	2.3
2.5	2.6	3.5	3.6	3.7	4	4.1	4.1
4.2	4.2	4.5	4.7	4.8			

   a. Use technology to create a histogram for the data with intervals 0–1, 1–2, and so on.
   b. Describe the shape of the distribution.
   c. Which interval has the highest frequency?

2. 

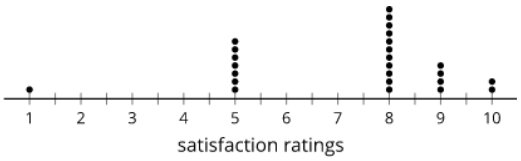
   a. Describe the shape of the distribution.

   b. How many values are represented by the histogram?

   c. Write a statistical question that could have produced the data set summarized in the histogram.

   (From Unit 1, Lesson 4)

3. The dot plot represents the distribution of satisfaction ratings for a landscaping company on a scale of 1 to 10. Twenty-five customers were surveyed. On average, what was the satisfaction rating of the landscaping company?

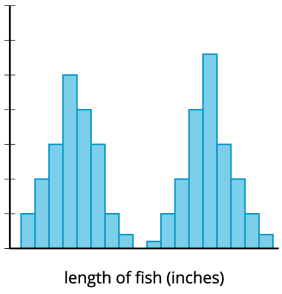


   (From Unit 1, Lesson 3)

4. This distribution shows the length in inches of fish caught and released from a nearby lake.

   a. Describe the shape of the distribution.

   b. Make an educated guess about what could cause the distribution to have this shape.



   (From Unit 1, Lesson 7)

Lesson 11: The Effect of Extremes

PREPARATION

Lesson Goals	Learning Targets
• Recognize the relationship between mean and median based on the shape of the distribution. • Understand the effects of extreme values on measures of center.	• I can describe how some data points can affect the mean and median. • I can use the shape of a distribution to compare the mean and median.

Lesson Narrative

The mathematical purpose of this lesson is to recognize a relationship between the shape of a distribution and the mean and median. Students will use histograms to investigate this relationship. Earlier in this unit, students created data displays so that the shape of the distribution is clear. This lesson connects to upcoming work because students will use the shape of the distribution and measures of center to make decisions about how to summarize data.

In this lesson, students make use of structure (MP7) and appropriate tools (MP5) to construct histograms of data that have prescribed measures of center.

The activities in this lesson work best when each student has access to technology that will easily compute measures of center and produce histograms because it will help students focus on understanding the relationship between extreme values and the measure of center without lengthy computations.

What is the main purpose of this lesson? What is the one thing you want your students to take away from this lesson?

Focus and Coherence

Building On	Addressing
NC.6.SP.3: Understand that both a measure of center and a description of variability should be considered when describing a numerical data set. **a.** Determine the measure of center of a data set and understand that it is a single number that summarizes all the values of that data set. ● Understand that a mean is a measure of center that represents a balance point or fair share of a data set and can be influenced by the presence of extreme values within the data set. ● Understand the median as a measure of center that is the numerical middle of an ordered data set. **NC.6.SP.5:** Summarize numerical data sets in relation to their context. **b.** Analyze center and variability by: ● Giving quantitative measures of center, describing variability, and any overall pattern, and noting any striking deviations. ● Justifying the appropriate choice of measures of center using the shape of the data distribution.	**NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots). **NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets. **NC.M1.S-ID.3:** Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Activity 1** (*20 minutes*)
  - Statistical technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device. The technology will be used for calculating measures of center and producing histograms.
- **Activity 2** (*5 minutes*)
- **Lesson Debrief** (*10 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L11 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

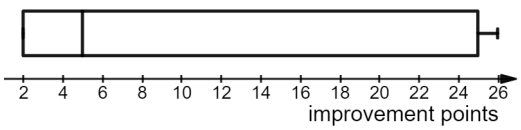
Building On: NC.6.SP.3.a; NC.6.SP.5.b

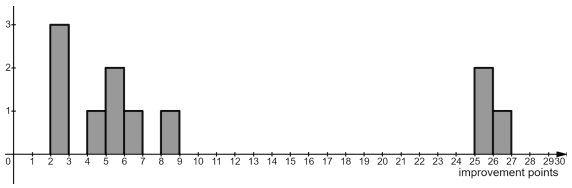
This bridge supports students who need to practice making a distinction between which measure of center is more representative of a data set. This will be useful later in this lesson when students manipulate data sets in order to observe the impact it has on the measures of center and shape of distribution.

Student Task Statement

A gym scores their clients on a scale of 1–100 and claims that they can help their clients improve on their exercises. After two months with a gym membership, eleven gym members are assessed to see how much they have improved on their exercises.

Here are two different data displays of the same data that represent how much the eleven gym members have improved.





1. What is the mean improvement among these members?
2. What is the median?
3. Which measure of center is a better representation of the members' improvement? Explain your reasoning.

DO THE MATH

PLANNING NOTES

Activity 1: Separated by Skew (*20 minutes*)

Instructional Routines: Co-Craft Questions (MLR5) - Responsive Strategy; Stronger and Clearer Each Time (MLR1)
Addressing: NC.M1.S-ID.1; NC.M1.S-ID.2**;** NC.M1.S-ID.3

The mathematical purpose of this activity is to help students understand how measures of center for distributions with different shapes are impacted by changes in the data. Students will create a histogram, then describe the shape of the distribution and find measures of center. They will investigate how the measures of center change when the data set changes. Students create and investigate a data set from a given a set of parameters including the shape of the distribution. Monitor for students using the correct terminology to describe the shape of the distribution.

This activity works best when each student has access to technology that computes measures of center and displays histograms easily because students will benefit from seeing the relationship in a dynamic way. If students don't have individual access, projecting the distributions would be helpful during Step 1.

This is the first time in the course that students might participate in a *Co-Craft Questions* routine.

CO-CRAFT QUESTIONS	***What Is This Routine?*** Students are presented with a picture, video, diagram, data display, or description of a situation, and their job is to generate one or more mathematical questions that could be asked about the situation. Students then share and compare their questions, as the teacher calls attention to questions that align with the content goals of the lesson. Finally, the "official" question or problem is revealed for students to work on. ***Why This Routine?*** *Co-Craft Questions* (MLR5) allows students to get inside of a context before feeling pressure to produce answers, and creates space for students to produce the language of mathematical questions themselves. Use this routine to spark curiosity about a new mathematical idea or representation, and to elicit everyday student language to brainstorm about the quantitative relationships that might be investigated. During this routine, students use conversation skills and develop meta-awareness of the language used in mathematical questions and problems.

Step 1

- Provide access to devices that can run Desmos or other statistical technology.
- Ask students to arrange themselve in small groups or use visibly random grouping, and instruct them to begin working on the activity.
- On problem 5, give each group one of these distribution descriptions. If needed, remind students of the words used to describe shapes of distributions: symmetric, skewed, bell-shaped, uniform, bimodal.
    1. uniform distribution with data between 4 and 12
    2. skewed right with most of the values at 10
    3. skewed left with most of the values at 10
    4. symmetric with most of the values at 4 and 16

RESPONSIVE STRATEGY

Before asking students to answer the given questions, display only the data and ask students to write down possible mathematical questions about this data distribution. Keep in mind students do not need to answer their created questions. Invite students to share and revise their questions with a partner, and then with the whole class. Record questions shared with the class in a public space. This helps students produce the language of mathematical questions as they begin to reason about the relationship between extreme values and the measure of center.

Co-Craft Questions (MLR5)

Advancing Student Thinking: Students may not yet be fluent in using Desmos to calculate measures of center and creating histograms. If needed, demonstrate these skills. Also, when students explore the effect of the additional values on mean and median they might not realize to reset to the original list of data for each case.

Student Task Statement

1. Use technology to create a histogram that represents the distribution of the data below.

   6 7 8 8 9 9 9 10 10 10 10 11 11 11 12 12 13 14

   Describe the shape of the distribution.
2. Find the mean and median of the data.
3. Find the mean and median of the data with two additional values included as described.

	Mean	Median
a. Add two values to the original data set that are greater than 14.		
b. Add two values to the original data set that are less than 6.		
c. Add one value that is greater than 14 and one value that is less than 6 to the original data set.		
d. Add the two values, 50 and 100, to the original data set.		

4. Share your work with your group. What do you notice is happening with the mean and median based on the additional values?
5. Change the values so that the distribution fits the description given to you by your teacher, then find the mean and median.
6. Find another group that created a distribution with a different description. Explain your work and listen to their explanation, then compare your measures of center.

Are You Ready For More?

The mean and the median are by far the most common measures of center for numerical data. There are other measures of center, though, that are sometimes used. For each measure of center, list some possible advantages and disadvantages. Be sure to consider how it is affected by extremes.

1. *Interquartile mean*: The mean of only those points between the first quartile and the third quartile.
2. *Midhinge*: The mean of the first quartile and the third quartile.
3. *Midrange*: The mean of the minimum and maximum value.
4. *Trimean:* The mean of the first quartile, the median, the median again, and the third quartile. So we are averaging four numbers as the median is counted twice.

Step 2

- Facilitate a whole-class discussion. The goal is to make sure that students understand that the median is the preferred measure of center when a distribution is skewed or if there are extreme values, and the mean is the preferred measure of center when a distribution is symmetric and there are no extreme values. Here are some questions for discussion:
  - "What do you notice and wonder about the mean and median for each of these distributions?" (I noticed that sometimes the median did not change and the mean did. I wondered what would happen if I added a value of 1,000 to the data set.)
  - "The mean best represents what is typical in the data for distributions that have what shape?" (the symmetric distributions)
  - "When is the median a better statistic to describe typical values?" (the skewed distributions)

RESPONSIVE STRATEGY

Break the class into small group discussion groups and then invite a representative from each group to report back to the whole class.

Supports accessibility for: Language; Social-emotional skills; Attention

Step 3

- Use the *Stronger and Clearer Each Time* routine focused on this question: "Why is the median a better statistic for skewed distributions?" (When you add extreme values to a data set, they tend to have a greater effect on the mean than the median.)

Why This Routine? *Stronger and Clearer Each Time* (MLR1) provides a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. The routine provides a purpose for student conversation and fortifies student output.

- Begin by asking students to write a first draft response to the question. Ready students to meet in pairs with several partners in a row, for a series of brief, 1–2 minute conversations about their initial responses to the prompt. In pairs, partners ask each other clarifying questions and provide additional ideas. Encourage students to take notes for themselves on feedback and useful ideas from each conversation.
- Display suggested prompts for feedback that will help students strengthen their ideas and clarify their language:
  - When you said _____, what did you mean?
  - I was confused when you said _____ because _____.
  - Can you give an example of what you mean?
  - What else do you think (or know) is true?
- After receiving feedback from two or three partners, ask each student to write a second draft response that is stronger and clearer than their initial response. Encourage students to borrow ideas and language from their partners as they write their own improved drafts.

DO THE MATH

PLANNING NOTES

Activity 2: Plots Matching Measures (*5 minutes*)

Addressing: NC.M1.S-ID.1; NC.M1.S-ID.2

The mathematical purpose of this activity is to recognize the relationship between measures of center and the shape of the distribution by creating and describing distributions with given measures of center. Listen for students using the terms symmetric, uniform, and skewed. When students create a histogram with a given mean and median using technology they are engaging in MP7 because they are using their understanding of the structure of the distribution to adjust individual data values to change the measures of center.

- Students can choose to work independently or with a partner to create the four histograms.

Student Task Statement

Use technology to create a histogram with at least 10 values for each of the conditions listed. Each histogram must have at least three values that are different.

1. A distribution that has both mean and median of 10
2. A distribution that has both mean and median of -15
3. A distribution that has a median of 2.5 and a mean greater than the median
4. A distribution that has a median of 5 and a median greater than the mean

DO THE MATH

PLANNING NOTES

Lesson Debrief (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)

Facilitate a whole-class discussion beginning with specific questions from Plots Matching Measures (Activity 2). The purpose of this discussion is for students to understand why the median is the preferred measure of center when a distribution is skewed or if there are extreme values, and the mean is the preferred measure of center when a distribution is symmetric and there are no extreme values.

This is the first time in the course that students will participate in *Discussion Supports*.

DISCUSSION SUPPORTS	***What Is This Routine?*** The teacher uses multi-modal strategies for helping students comprehend and generate language and ideas, such as sentence frames, word walls, images and videos, revoicing, choral response, gesture, and graphic organizers. The strategies can be combined and used together with any of the other routines. ***Why This Routine?*** *Discussion Supports* (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use *Discussion Supports* to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

- For each description, select two or three groups to share their histograms. If students are using the Desmos calculator, groups can share the link to their histogram which can then be displayed for the entire class during the discussion.
- Use *Discussion Supports* as groups share their histograms with the whole class, and revoice student ideas for determining the appropriate measure of center based on the shape of the distribution. Be sure to amplify mathematical uses of language by restating a statement as a question in order to clarify, apply appropriate language, and involve more students. For example, “Can someone else explain why the median is used for skewed data?” Press for details in students’ explanations by requesting students to elaborate on an idea or give an example from their data representation.

Here are some questions to draw out the relationship between measures of center and the shape of the distribution.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- “For the first and second histogram, what do the distribution shapes have in common? Why do we choose the mean as the more appropriate measure of center?” (Symmetric. The mean of a set of data gives equal importance to each value to find the center, so it is a preferred measure of center when it accurately represents typical values for data.)
- “What do the shapes of the histograms have in common when the mean is greater than the median?” (Skewed right.)
- “What information does the shape of the skewed distributions tell you about the median and mean?” (When distributions are skewed right, they will likely have a mean that is greater than the median because the values to the right disproportionately impact the mean. When distributions are skewed left, they will likely have a mean that is less than the median because the values to the left disproportionately impact the mean.)
- “Why is the median preferred to the mean for skewed data?” (The values way to the right (or left) in skewed data have a greater effect on the mean, so the median is preferred to better reflect the typical values.)
- “When an extreme value is present, why is the median preferred to the mean?” (Extreme values have a greater effect on the mean than the median so the median is preferred.)

PLANNING NOTES

Student Lesson Summary and Glossary

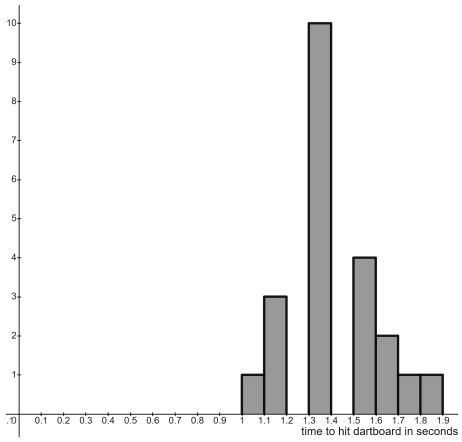
Have you heard the word **statistic** before? You have been using different statistics in all the data that you have examined.

> **Statistic**: A quantity that is calculated from sample data, such as mean or median.

How do you decide when to use the mean or median to describe the center of a data set?

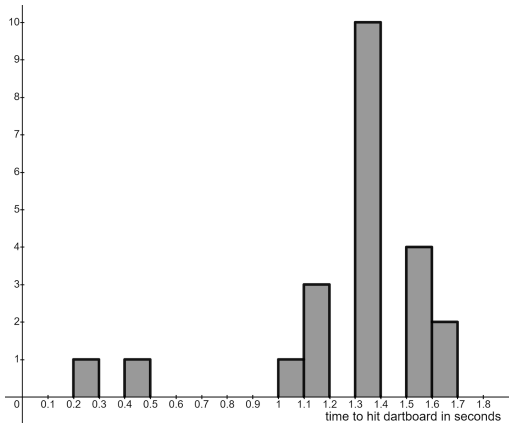
- The mean gives equal importance to each value. The mean usually represents the typical values well when the data sets have a symmetric distribution. On the other hand, the mean can be greatly affected by changes to even a single value.
- The median tells you the middle value in the data set, so changes to a single value usually do not affect the median much. Because of this, the median is more appropriate for data sets that are not very symmetric.

Here is a histogram showing the amount of time a dart takes to hit a target in seconds. The data produce a symmetric distribution.



Since the distribution is symmetric, the mean should be used to describe a typical value of the data.

Here is a histogram using the same data, but with two of the values changed, resulting in a skewed distribution.



Since the distribution is skewed, the median should be used to describe a typical value of the data. The extreme values at the lower end make the mean lower than most values in the data set. The median is more resistant to those extreme values, so it is the preferred measure of center.

The distribution above was skewed left, making the mean lower than the median. Likewise, if a distribution is skewed right, the mean will almost always be higher than the median.

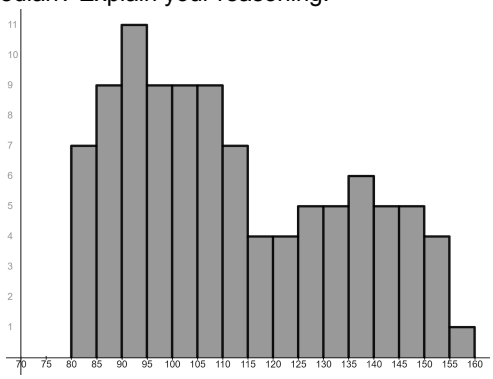
Cool-down: Shape and Statistics (*5 minutes*)

Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

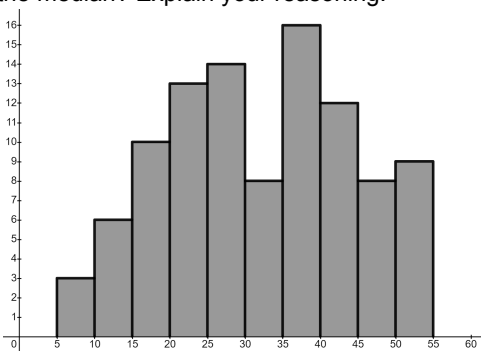
Cool-down Guidance: Points to Emphasize
If students struggle to recognize how the mean and median are affected by extreme values, Activity 1 in Lesson 13 provides an opportunity to highlight mean and median and which is the appropriate measure of center. The matching activity provides many examples to highlight whether the mean or the median is greater in a skewed distribution and why.

Cool-down

1. Is the mean greater than, less than, or equal to the median? Explain your reasoning.



2. Is the mean greater than, less than, or equal to the median? Explain your reasoning.



Student Reflection:

Give an example of where you might see something you learned today in the real world.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

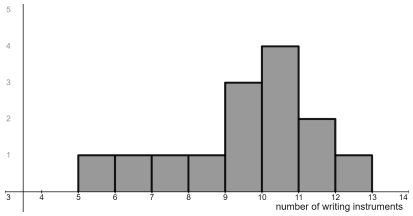
Students shared their thinking multiple times in this lesson. What have you noticed about the language students use? What support can you offer to students who struggle to communicate their ideas orally?

Practice Problems

1. Select **all** the distribution shapes for which it is most often appropriate to use the mean.
    a. bell-shaped
    b. bimodal
    c. skewed
    d. symmetric
    e. uniform

2. For which distribution shape is it usually appropriate to use the median when summarizing the data?
   a. bell-shaped
   b. skewed
   c. symmetric
   d. uniform

3. The number of writing instruments in some teachers' desks is displayed in the dot plot. Which is greater, the mean or the median? Explain your reasoning using the shape of the distribution.



4. A student has these scores on their assignments. The teacher is considering dropping the lowest score. What effect does eliminating the lowest value, 0, from the data set have on the mean and median?

   0, 40, 60, 70, 75, 80, 85, 95, 95, 100

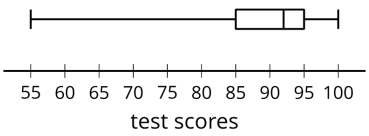
   (From Unit 1, Lessons 9 & 10)

5. The data set 2, 2, 4, 4, 5, 5, 6, 7, 9, 15 represents the number of books purchased by the first 10 customers in a bookstore.

   a. What is the five-number summary?
   b. When the maximum, 15, is removed from the data set, what is the five-number summary?

   (From Unit 1, Lessons 9 & 10)

6. The box plot summarizes the test scores for 100 students:

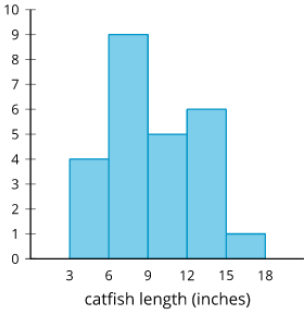


   Which term best describes the shape of the distribution?

   a. bell-shaped
   b. skewed right
   c. skewed left
   d. symmetric

   (From Unit 1, Lesson 7)

7. The histogram represents the distribution of lengths, in inches, of 25 catfish caught in a lake.
   a. If possible, find the mean. If not possible, explain why not.
   b. If possible, find the median. If not possible, explain why not.
   c. Were any of the fish caught 12 inches long?
   d. Were any of the fish caught 19 inches long?



   (From Unit 1, Lesson 5)

8. For five days, Tyler has recorded how long his walks to school take in minutes. The mean for his data is 11 minutes.[1]

   a. Without calculating, predict if each of the data sets shown could be Tyler's. Explain your reasoning.
      - data set A (11, 8, 7, 9, 8)
      - data set B (12, 7, 13, 9, 14)
      - data set C (11, 20, 6, 9, 10)
      - data set D (8, 10, 9, 11, 11)

   b. Determine which data set is Tyler's. Explain how you know.

   (Addressing NC.6.SP.3.a)

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 12: Standard Deviation

PREPARATION

Lesson Goals	Learning Targets
• Comprehend (in spoken and written language) standard deviation as a measure of variability. • Use technology to compute standard deviation.	• I can describe standard deviation as a measure of variability. • I can use technology to compute standard deviation.

Lesson Narrative

The mathematical purpose of this lesson is to introduce **standard deviation** and understand that it is a measure of variability. Previously, students understood the meaning of IQR as a measure of variability associated with median; standard deviation fits into their understanding as another measure of variability. Future lessons build on this lesson when students interpret standard deviation in context. Standard deviation is a measure of variability calculated by:

- Finding the square of the distance from the mean to each value.
- Finding the sum of these square distances and dividing by n (the number of values in the data set).
- Finally, finding the square root of this sum.

In Math 1, students will use technology to compute the standard deviation.

When students manipulate data to achieve various specified measures of center or variability they are engaging in MP2 because they have to make use of the structure underlying standard deviation as a measure of variability.

Note that in this unit, all standard deviations refer to the population standard deviation (σ) calculation rather than the sample standard deviation (s).

In what ways will you encourage students to persevere in this lesson?

Focus and Coherence

Building On	Addressing
NC.M1.S-ID.1: Use technology to represent data with plots on the real number line (histograms and box plots).	**NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets. **NC.M1.S-ID.3:** Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Agenda, Materials, and Preparation

- **Warm-up** (*5 minutes*)
  - Statistical technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device. The technology will be used for calculating standard deviation and mean.
  - Desmos Timer App: http://bit.ly/desmostimer
- **Activity 1** (*15 minutes*)
  - *Responsive Strategy*: Investigating Standard Deviation graphic organizer (print as needed)
- **Activity 2** (*15 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L12 Cool-down (print 1 copy per student)

LESSON

Warm-up: Estimating Time (*5 minutes*)

Building On: NC.M1.S-ID.1	**Building Towards**: NC.M1.S-ID.2

The purpose of this warm-up is to build an understanding of how values in a data set can vary from the mean. This will be useful when students explore standard deviation in a later activity. This warm-up prompts students to consider how their own estimation of time varies from the goal of 10 seconds. This leads to a discussion about how the estimates vary. You could collect data from students and create a histogram to analyze and discuss. A sample data set of 100 students has been provided.

Step 1

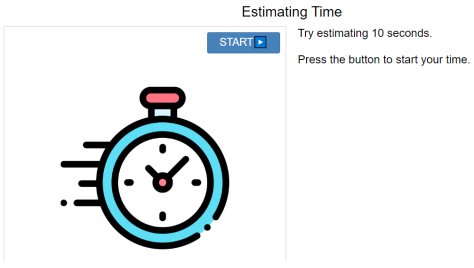
- Ask students to form pairs or use visibly random grouping.
- If students are using the Desmos app, provide instructions on how to access and use it. If students are using other methods for tracking the time you may need to provide them with alternative instructions. (Link to app: http://bit.ly/desmostimer)
- Tell students to think quietly about their answers to the questions for about a minute.
- Have students discuss with their partner and record their responses.

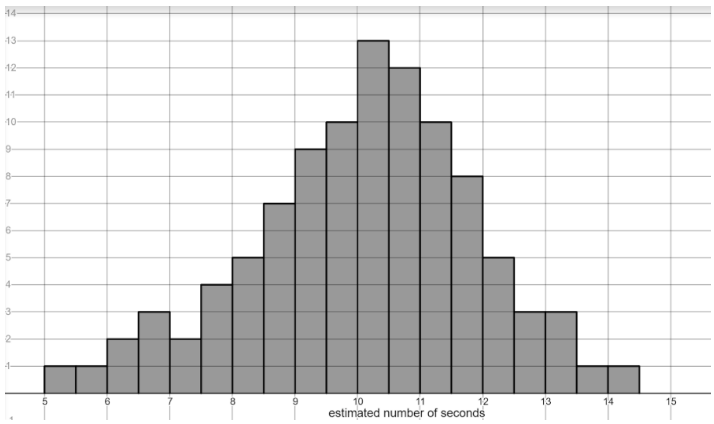
Student Task Statement

The passing of time is constant, but our estimation of how much time has passed can vary. How close are we at estimating 10 seconds?

Open the Estimating Time app. When ready, press the button to start the timer. When you believe 10 seconds has passed press the button again to stop the timer. Record the amount of time that actually passed.

1. How close was your estimate to being 10 seconds?
2. This histogram shows the estimates of 100 students. *(See next page.)*





- Describe the shape.
- Estimate the center of the data set and mark it on the horizontal axis.
- Mark on the histogram:
  - the estimates that are the best
  - the estimates that are close
  - the estimates that are not so close

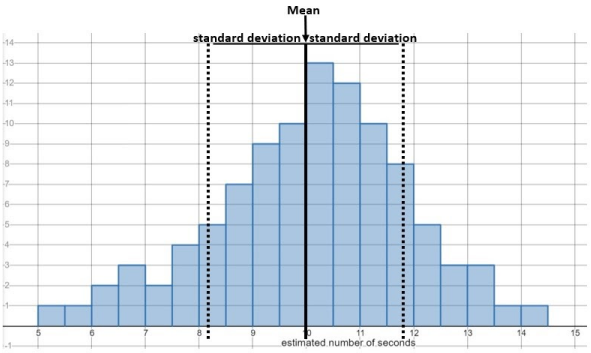
3. Consider the estimates that are close: some will be too high and some will be too low. Approximately how far from the mean are these values?

Step 2

- Facilitate a whole-class discussion. The purpose of the discussion is for students to understand how values in a data set can vary from the mean. The discussion also provides an opportunity for students to estimate the standard deviation by considering values that are close to the mean.

  Here are some questions for discussion:

  - How could you use the histogram to determine that students were trying to estimate 10 seconds? (The distribution has a mean of 10 seconds.)
  - How did you calculate how close your estimate was to 10 seconds? (I subtracted my estimate from 10 seconds. This difference is referred to as a deviation from the mean. Since not all estimates were 10 seconds there is variation in the estimates.)
  - How did you decide which estimates were close? (I looked for the number of seconds that weren't too far from the center. I decided that the estimates between 8 seconds and 12 seconds were close to 10 seconds.)



  Tell students that the deviations from the mean are used in calculating a measure of variability called the standard deviation. The value of the standard deviation for this distribution is approximately 1.8 seconds. This means that on average the estimates were between 1.8 seconds below the mean and 1.8 seconds above the mean. Illustrate this by annotating the histogram.

DO THE MATH

PLANNING NOTES

Activity 1: Investigating Standard Deviation (*15 minutes*)

Instructional Routine: Co-Craft Questions (MLR5) - Responsive Strategy	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

The purpose of this activity is to let students investigate the standard deviation using different data sets. The goal is for students to make conjectures about what standard deviation measures and how relative size of the standard deviation can be estimated from the shape of the distribution. In particular, students should recognize that adding or subtracting the same amount from each value in the data set will change the mean by that amount, but the standard deviation remains unchanged. Multiplying or dividing each value in the data set by the same amount scales both the mean and standard deviation by that amount.

RESPONSIVE STRATEGY

Display only the first line of this task ("Use technology to find the mean and the standard deviation for the data in the dot plots") and the dot plots, and ask pairs of students to write possible mathematical questions about the distributions. Then, invite pairs to share their questions with the class. This offers a way to formatively assess the language students are comfortable producing at this point in the unit about data, dot plots, mean, and standard deviation.

Co-Craft Questions (MLR5)

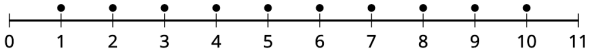
Step 1

- Demonstrate how to find standard deviation and mean using Desmos.
  - Enter the data as a *list* in Desmos. Access the standard deviation available from the "Stats" section of the "functions" menu on the keypad or type directly into an expression using a keyboard. Use the *listname* as the argument. For example, stdevp(L) will return the population standard deviation for the list of data named L.
  - Also, demonstrate how when adding, changing, and removing data points from a data set, the standard deviation and mean will recalculate.
- Ask students to form pairs or use visibly random grouping. Give students time to work through the first two questions, followed by a whole-class discussion.

Student Task Statement

1. Use technology to find the mean and the standard deviation for the data in the dot plots.

Partner 1 Dot plots:

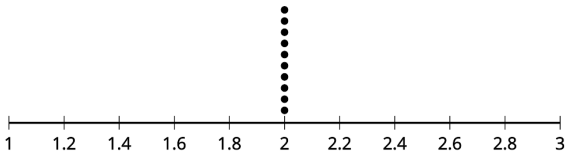


Mean: _____

Standard deviation: _____

Mean: _____

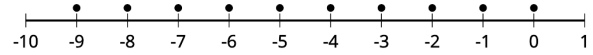
Standard deviation: _____



Mean: _____

Standard deviation: _____

Partner 2 Dot plots:



Mean: _____

Standard deviation: _____

Mean: _____

Standard deviation: _____

Mean: _____

Standard deviation: _____

2. What do you notice about the mean and standard deviation you and your partner found for the three dot plots?

Are You Ready For More?

Invent some data that fits the conditions. Be prepared to share your data set and reasoning for choice of values.

1.
   a. 10 numbers with a standard deviation equal to the standard deviation of your first dot plot with a mean of 6.
   b. 10 numbers with a standard deviation equal to the standard deviation of your first dot plot with a mean of 12.
2.
   a. 10 numbers with a standard deviation three times greater than the data in the first row.
   b. 10 numbers with a standard deviation four times greater than the data in the first row.

Step 2

- Facilitate a whole-class discussion. The purpose of this discussion is to understand that the standard deviation is a measure of variability related to the mean of the data set. The discussion also provides an opportunity for students to discuss what they notice and wonder about the mean and standard deviation.

Here are some questions for discussion.

- "What do you think standard deviation measures? Why do you think that?" (I think that it measures variability. When all the values were the same it was zero. When the data were more spread out, the standard deviation was larger than when the data were closer together.)

RESPONSIVE STRATEGY

Provide students with a graphic organizer with standard deviation at the center, connecting to it all the related concepts mentioned during the discussion.

Supports accessibility for: Language; Organization

- "Why is the standard deviation the same for {1,2,3,4,5} and {-2,-1,0,1,2}?" (For each data set: 1) the values to the left of the mean are a distance of 2 and 1 from the mean and the values to the right of the mean are a distance 2 and 1 from the mean, and 2) the middle value is 0 away from the mean. This results in the standard deviation being the same.)
- "Why is the standard deviation different for {-4, -2, 0, 2, 4} and {-4, -3, -2, -1, 0}?" (The values for the first data set are twice the distance from the mean as the values in the second data set. That makes the standard deviation for the first set greater than the standard deviation of the second data set.)
- "When is the standard deviation equal to zero?" (It is zero when all the values are the same as each other or when there is no variability.)
- "How did using technology help or hinder your mathematical thinking about standard deviation and mean?" (It really helped me because I did not get bogged down with the calculations and I could look for patterns in the data, the data displays, and the statistics.)

DO THE MATH

PLANNING NOTES

Activity 2: Investigating Variability (*15 minutes*)

Instructional Routine: Stronger and Clearer Each Time (MLR1) - Responsive Strategy	
Building On: NC.M1.S-ID.1; NC.M1.S-ID.2	**Addressing:** NC.M1.S-ID.3

The mathematical purpose of this activity is to let students investigate how the standard deviation and other measures of variability change when you add, change, or remove values in a data set. Monitor for students mentioning the concepts of shape, variability, and center. This activity works best when each student has access to statistical technology because it would take too long to compute the statistics otherwise. If students don't have individual access, project the statistical technology during Step 1.

Advancing Student Thinking: Students who compute a different standard deviation may be using the sample standard deviation statistic. Tell these students to use the value for σ rather than s for computations in this unit. In Desmos, the population standard deviation is **stdevp**.

RESPONSIVE STRATEGIES

Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "I notice that the mean is affected when…." or "Standard deviation will always be affected because…."

Supports accessibility for:
Language; Organization

Step 1

- Give students time to work through the questions then pause for a brief whole-class discussion.

Student Task Statement

Begin with this data set: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

1. Use technology to find the mean, standard deviation, and median. Calculate the interquartile range (IQR).
2. How do the standard deviation and mean change when you remove the greatest value from the data set? How do they change if you add a value to the data set that is twice the greatest value?
3. What do you predict will happen to the standard deviation and mean when you remove the least value from the data set? Check to see if your prediction was correct.
4. What happens to the standard deviation and mean when you add a value to the data set equal to the mean? Add a second value equal to the mean. What happens?
5. Add, change, and remove values from the data set to answer the question: What appears to change more easily, the standard deviation or the interquartile range? Explain your reasoning.

Are You Ready For More?

How is the standard deviation calculated?

1. Using the original data set, calculate the deviation of each point from the mean by subtracting the mean from each data point.
2. If we just tried to take a mean of those deviations, what would we get?
3. There are two common ways to turn negative values into more useful positive values: take the absolute value or square the value. To find the standard deviation we square each of the deviations, then find the mean of those numbers. Then finally take the square root of that mean. Compute the standard deviation of the original data set.

Step 2

- Facilitate a whole-class discussion. The purpose of this discussion is to talk about how the standard deviation and other measures of variability change when you add, change, or remove values in a data set.The goal of this discussion is to make sure that students understand that the standard deviation is a measure of variability that uses all values from the data set. Its behavior is similar to that of the mean in that it reacts to changes in the data values.
  - Discuss how the standard deviation is impacted by the addition and removal of values in the data set. The standard deviation decreases when values on either end (low or high) are removed because there is a smaller spread. The data in the distribution display less variability. The standard deviation increases when values are added on either end (low or high) because there is a larger spread. The data in the distribution display more variability.
  - Discuss how the standard deviation decreases slightly when adding values to the data set that are equal to the mean. As the values become more concentrated in the center the variability decreases.

RESPONSIVE STRATEGY

Use this routine to give students a structured opportunity to revise and refine their response to the question: "What appears to change more easily, the standard deviation or the interquartile range?". Ask each student to meet with 2–3 different partners in a row for feedback. Provide students with prompts for feedback that will help teams strengthen their ideas and clarify their language. For example, "Can you explain how...?", "Could you expand on....", etc. Encourage students to borrow ideas and language from each partner to strengthen their revised response.

Stronger and Clearer Each Time (MLR1)

- Discuss which measure of variability is impacted more when changing the values of a data set. The standard deviation changes more because it uses all values in the data set. The IQR is usually not impacted by changes to values on either end as it uses the middle half of the data.
- Add standard deviation to the display of measures of center and measures of variability created in Lesson 8. (See Student Lesson Summary and Glossary as an example.)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

Facilitate a lesson debrief.

Choose whether students should first have an opportunity to reflect in their workbooks or talk through the questions with a partner. Determine what questions will be prioritized in the full class discussion.

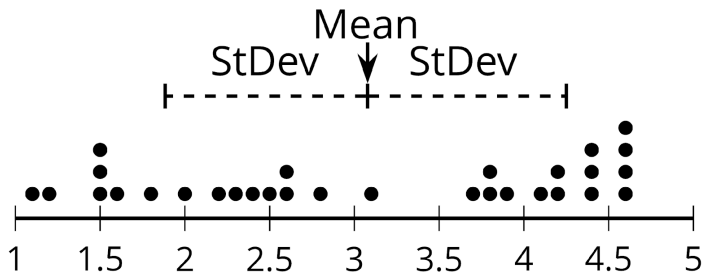
- "What does the standard deviation measure? How do you know?" (It measures variability. The larger the spread the higher the standard deviation. The smaller the spread the smaller the standard deviation.)
- "The standard deviation is calculated using the mean. Do you think it is more appropriate to use with symmetric or skewed data sets?" (Symmetric, because it is calculated using the mean.)
- How does standard deviation compare and contrast with IQR?" (Like the IQR, the standard deviation is a measure of variability. Unlike the IQR, standard deviation is based on the mean not the median.)
- "One data set has a standard deviation of 5 and another data set has a standard deviation of 10. What does this tell you about the distribution of each data set?" (Standard deviation is a measure of variability so it tells you how spread apart the data are. The second data set shows greater variability than the first data set.)

PLANNING NOTES

Student Lesson Summary and Glossary

We can describe the variability of a distribution using the **standard deviation**. The standard deviation is a measure of variability that tells you how spread out the data points are from the mean. The standard deviation is large when the data are more spread out, and the standard deviation is small when the data are close together.

Since standard deviation is a measure of variability, two data sets with the same shape will have the same standard deviation. For instance, the data sets {1, 3, 5} and {4, 6, 8} have the same standard deviation because the distances from the mean for both data sets are 2, 0, and 2. Unlike IQR, the standard deviation changes a lot when you add or remove certain values from a data set.



Standard deviation: A measure of variability based on how spread out the data points are from the mean.

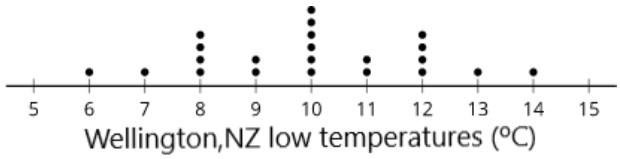
Cool-down: True or False: Reasoning with Standard Deviation (*5 minutes*)

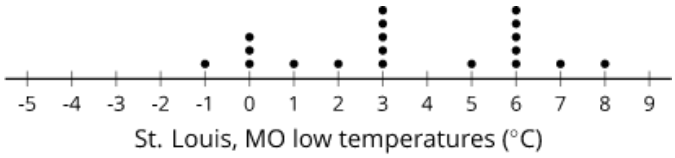
Addressing: NC.M1.S-ID.2

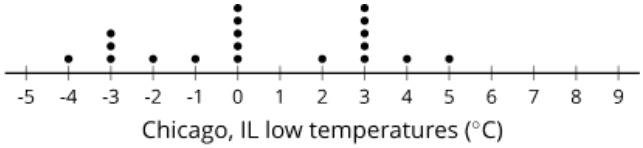
Cool-down Guidance: More Chances
There will be more opportunities to discuss standard deviation. Use the results of the cool-down to determine what to emphasize. One key idea to emphasize is that data with the same shape (in the same scale) have the same variability.

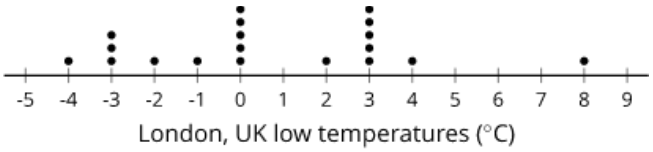
Cool-down

The low temperature in degrees Celsius for some cities on the same days in March are recorded in the dot plots.









Decide if each statement is true or false. Explain your reasoning.

1. The standard deviation of Wellington's temperatures is about 4 degrees Celsius.
2. The standard deviation of St. Louis's temperatures is equal to the standard deviation of Chicago's temperatures.
3. The standard deviation of Chicago's temperatures is less than the standard deviation of London's temperatures.

Student Reflection:

What is at least one thing you did differently within the past week to improve your skills or understandings in Math 1?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

Reflect on your experience with one of the instructional and language routines in the curriculum. What moves or questions have improved the learning for each or your students during this routine? What improvements would you make next time?

Practice Problems

1. The shoe size for all the pairs of shoes in a person's closet are recorded.

   7 7 7 7 7 7 7 7 7 7

   a. What is the mean?

   b. What is the standard deviation?

2. Here is a data set:

   1 2 3 3 4 4 4 4 5 5 6 7

   a. What happens to the mean and standard deviation of the data set when the 7 is changed to a 70?

   b. For the data set with the value of 70, why would the median be a better choice for the measure of center than the mean?

3. Here is a data set with a mean 57 of and a standard deviation of 6.9:

44 54 57 57 60 61 66

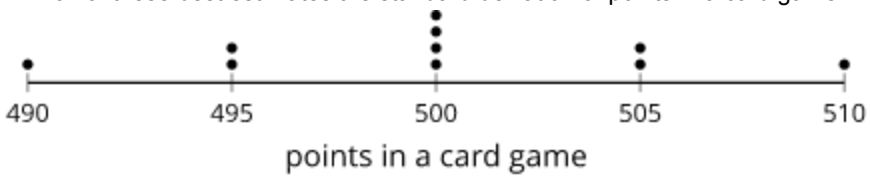
a. Each value in this new data set increases each original value by three. Without calculating, find the new mean and standard deviation of this data set:

47 57 60 60 63 64 69

b. Each value in this new data set is twice as big as the values in the original data set. Without calculating, find the new mean and standard deviation of this data set:

88 108 114 114 120 122 132

4. Which of these best estimates the standard deviation of points in a card game?



points in a card game

a. 5 points
b. 20 points
c. 50 points
d. 500 points

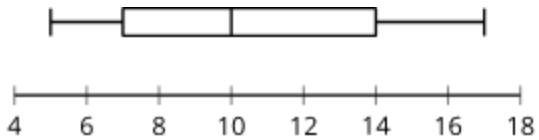
5. Select **all** the distribution shapes for which the mean and median *must be* about the same.

a. bell-shaped
b. bimodal
c. skewed
d. symmetric
e. uniform

(From Unit 1, Lesson 11)

6. What is the IQR?

a. 5 branches
b. 7 branches
c. 10 branches
d. 12 branches



number of branches on 10 year old trees

(From Unit 1, Lesson 13)

7. The data represent the number of cans collected by different classes for a service project.

12 14 22 14 18 23 42 13 9 19 22 14

a. Find the mean.

b. Find the median.

c. Eliminate the greatest value, 42, from the data set. Explain how the measures of center change.

(From Unit 1, Lesson 8)

Lesson 13: Comparing and Contrasting Data Distributions

PREPARATION

Lesson Goal	Learning Target
• Interpret (orally) a data set with greater standard deviation or IQR as having greater variability.	• I can arrange data sets in order of variability given graphic representations.

Lesson Narrative

The mathematical purpose of this lesson is to compare data sets with different measures of variability and to interpret data sets with greater standard deviations or IQRs as having greater variability. Students make connections between different data displays and measures of center and measures of variability. This lesson builds on previous work by using what students learned when creating data displays and determining which measure of center is appropriate.

When students match data displays with descriptions of shape, measures of center, or measures of variability, they are engaging in MP2 because they are having to make sense of the data display and the corresponding description or measure. When students participate in a math talk about finding the mean, they have an opportunity to notice and make use of the symmetric structure (MP7) of the values to determine the mean. Additionally, students need to be precise in their word choice and use of language (MP6). In Activity 2, students trade roles explaining their thinking and listening, providing opportunities to explain their reasoning and critique the reasoning of others (MP3).

Which Standards for Mathematical Practice (SMPs) do you anticipate students engaging in during this lesson? How will you support them?

Focus and Coherence

Building On	Addressing	Building Towards
NC.6.SP.3: Understand that both a measure of center and a description of variability should be considered when describing a numerical data set. **b**.Understand that describing the variability of a data set is needed to distinguish between data sets in the same scale, by comparing graphical representations of different data sets in the same scale that have similar measures of center, but different spreads. **NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots).	**NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.	**NC.M1.S-ID.3:** Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*5 minutes*)
- **Activity 1** (*15 minutes*)
  - Describing Data Distributions cards (print 1 copy for every 2 students and cut in advance)
- **Activity 2** (*15 minutes*)
  - *Responsive Strategy*: Visual Variability and Statistics Venn diagram (print as needed)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L13 Cool-down (print 1 copy per student)

LESSON

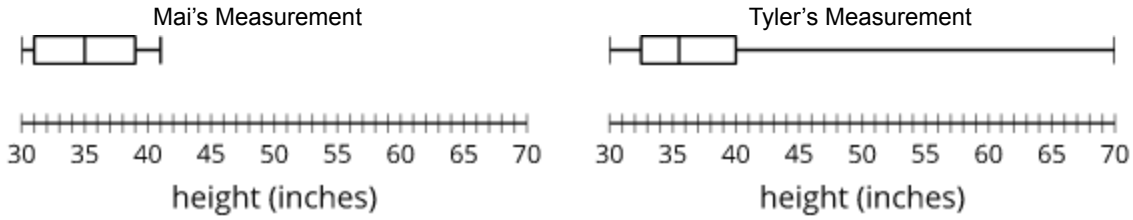
Bridge (*Optional, 5 minutes*)

Building On: NC.6.SP.3b

This bridge supports students in using boxplots to interpret variability. While students may notice and wonder many things about these data displays, the similarity between the measures of center and the differences in variability are key components in preparing for work later in this lesson. This bridge will be useful when students match data displays with measures of center and variability.

Student Task Statement

Mai and Tyler both visit the same preschool classroom and measure the heights of people in the room in inches. The summary of their results are shown in the box plot and statistics. What do you notice? What do you wonder?



DO THE MATH

PLANNING NOTES

Warm-up: Math Talk: Mean (*5 minutes*)

Instructional Routines: Math Talk; Discussion Supports (MLR8) - Responsive Strategy
Addressing: NC.M1.S-ID.2

This is the first time in the course that students will participate in a *Math Talk* routine.

MATH TALK

What Is This Routine? In these warm-ups, one problem is displayed at a time. Students are given a few moments to quietly think and give a signal when they have an answer and a strategy. The teacher selects students to share different strategies for each problem, asking, "Who thought about it a different way?" Their explanations are recorded for all to see. Students might be pressed to provide more details about why they decided to approach a problem a certain way. It may not be possible to share every possible strategy in the given time—the teacher may only gather two or three distinctive strategies per problem. Problems are purposefully chosen to elicit different approaches, often in a way that builds from one problem to the next.

Why This Routine? A *Math Talk* builds fluency by encouraging students to think about the numbers, shapes, or algebraic expressions and rely on what they know about structure, patterns, and properties of operations to mentally solve a problem. While participating in these activities, there is a natural need for students to be precise in their word choice and use of language (MP6). Additionally, a Math Talk often provides opportunities to notice and make use of structure (MP7).

The purpose of this Math Talk is to expand students' strategies for finding a mean beyond following an algorithm to reasoning that the mean of the values in a symmetric data set is the middle value. The third item is designed to illustrate that this technique only works for symmetric data sets. These understandings help students develop fluency and will be helpful later in this lesson when students will need to use symmetry to match a mean to the distribution.

Step 1

- Display one problem at a time.
- Give students quiet think time for each problem.
- Ask them to give a signal when they have an answer and a strategy.
- Keep all problems displayed throughout the talk.

RESPONSIVE STRATEGY

To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Advancing Student Thinking: If students struggle to use symmetry as a method for finding the mean, consider asking them to find the mean for the values: 1, 2, 3, 4, 5.

Student Task Statement

Evaluate the mean of each data set mentally.

- 27, 30, 33
- 61, 71, 81, 91, 101
- 0, 100, 100, 100, 100
- 0, 5, 6, 7, 12

Step 2

- Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:
  - "Who can restate ___'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone solve the problem in a different way?"
  - "Does anyone want to add on to ___'s strategy?"
  - "Do you agree or disagree? Why?"

RESPONSIVE STRATEGY

Display sentence frames to support students when they explain their strategy. For example, "First, I ___ because . . ." or "I noticed ___ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Discussion Supports (MLR8)

Monitoring Tip: Monitor for students who:
- use the standard algorithm for finding mean (sum and divide)
- use the symmetry of the data set

Although all correct methods for solving for the mean are valid, highlight the use of symmetry in the data. Since symmetric distributions have a mean in the center of the data, that can be used to quickly discover the mean.

DO THE MATH

PLANNING NOTES

Activity 1: Describing Data Distributions (*15 minutes*)

Instructional Routines: Take Turns; Card Sort; Collect and Display (MLR2)		
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2	**Building Towards:** NC.M1.S-ID.3

In this activity, students take turns with a partner matching data from the Describing Data Distribution cards with characteristics of a distribution and determine what measure of center is most appropriate for the data. Students trade roles explaining their thinking and listening, providing opportunities to explain their reasoning and critique the reasoning of others (MP3). Reference the blackline master for the full visual of the matching activity.

Step 1

- Ready students for the *Take Turns* routine by having them arrange themselves into pairs or use visibly random grouping.

Why This Routine? Building in an expectation, through the *Take Turns* routine, that students explain the rationale for their choices and listen to another's rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others' reasoning (MP3).

- Demonstrate the *Card Sort* routine by showing how to set up and find matches.
  - Mix up the cards and place them face-up.
  - Point out that the cards contain either a data display or a written statement.
  - Select one of each type of card and then explain to your partner why you think the cards do or do not match.
- Demonstrate productive ways to agree or disagree: for example, by explaining your mathematical thinking or asking clarifying questions.
- Give each group a set of cut-up cards for matching.
- Ask students to pause after completing the matching for a whole-class discussion.

RESPONSIVE STRATEGY

Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, "This is a match because ____." or "I would use mean or median for the center of data because ____."

Supports accessibility for: Social-emotional skills; Organization; Language

Why This Routine? A *Card Sort* provides opportunities to attend to mathematical connections using representations that are already created, instead of expending time and effort generating representations. It gives students opportunities to analyze representations, statements, and structures closely and make connections (MP2, MP7).

Step 2

Much discussion takes place between partners. Once all groups have completed the matching, discuss the following:

- "Which matches were tricky? Explain why." (The box plot F was tricky because I had to use the process of elimination to figure out that it was the one that was uniform.)
- "Did you need to make adjustments in your matches? What might have caused an error? What adjustments were made?" (Yes. I realized that I thought incorrectly that skewed left meant that most of the data were on the left. However, I learned that skewed left means that there are data to the left of where most of the data are located.)
- "Can you determine the median using only a histogram? Why or why not?" (No, but you can determine the interval that contains the median.)
- "Can you determine if a distribution is uniform from a box plot? Why or why not?" (No. You can determine that the data could possibly be symmetric based on the five number summary, but beyond that you would not be able to know that the data are uniform using only a box plot.)

Step 3

- Students continue the activity by working on the second question. Be sure to name for students that the appropriate measure of center may not be the one given on the cards.

Step 4

The purpose of question 2 is to discuss the relationship between mean and median based on the shape of the distribution and to make the connection to measures of variability. Ask:

- "If the mean is the appropriate measure of center, should we use the standard deviation or the IQR to measure variability? Explain how you know." (Standard deviation, because the standard deviation is calculated using the mean. Standard deviation, because it represents a typical distance from the mean.)
- "If the median is the appropriate measure of center, should we use the standard deviation or the IQR to measure variability? Explain how you know." (IQR, because IQR is based on the five-number summary. IQR, because you use the median to find the quartiles, which are then used to calculate IQR.)

Use the *Collect and Display* routine to invite students to investigate another pair's work and say what worked well in their approach. Listen for and capture any comments about the use of data, medians, and means to compare the different distributions with the written statements, and any language students use to describe the shape of the distribution (skewed, symmetric, or uniform). Write down student language on the board or chart paper for all to see, and continue to build this display of student language in the next activity.

Student Task Statement

1. Your teacher will give you a set of cards. Take turns with your partner to match a data display with a written statement.
    a. For each match that you find, explain to your partner how you know it's a match.
    b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. After matching, determine if the mean or median is more appropriate for describing the center of the data set based on the shape of the distribution. Discuss your reasoning with your partner. If it is not given, calculate (if possible) or estimate the appropriate measure of center. Be prepared to explain your reasoning.

DO THE MATH

PLANNING NOTES

Activity 2: Visual Variability and Statistics (*15 minutes*)

Instructional Routine: Collect and Display (MLR2)	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

This activity prompts students to compare variability in several data sets by analyzing box plots and dot plots. Some students may reason about variability by observing the shapes and features of the data displays. Others may try to quantify the variability by finding the IQR from each box plot, or by estimating the standard deviation from each dot plot. Look for students who approach the task quantitatively.

Step 1

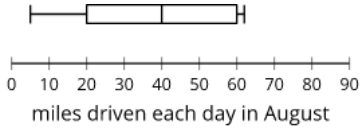
- Ask students to arrange themselves in pairs or use visibly random grouping.
- Students should spend 2 minutes working independently and then 3 minutes working through the questions with their partner.
- Facilitate a whole-group discussion.

***Advancing Student Thinking*:** Students may have forgotten what variability means or which statistic to use to determine the variability in a data set. Refer them to previous work or ask them what measure is useful in determining a data set's tendency to have different values.

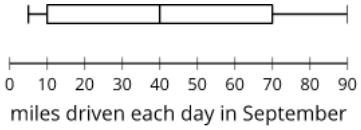
Student Task Statement

Each box plot summarizes the number of miles driven each day for 30 days in each month. The box plots represent, in order, the months of August, September, October, November, and December.

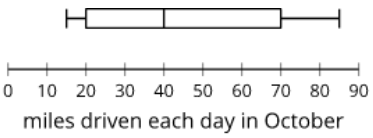
a.



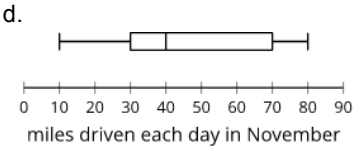
b.



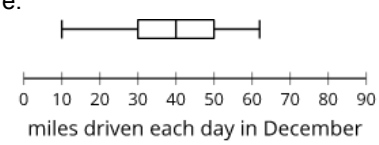
c.



d.

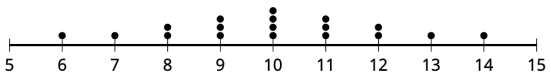


e.

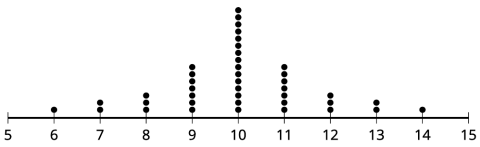


1. The five box plots have the same median. Explain why the median is more appropriate for describing the center of the data set than the mean for these distributions.
2. Arrange the box plots in order of least variability to greatest variability. Check with another group to see if they agree.
3. The five dot plots have the same mean. Explain why the mean is more appropriate for describing the center of the data set than the median.
4. Arrange the dot plots in order of least variability to greatest variability. Check with another group to see if they agree.

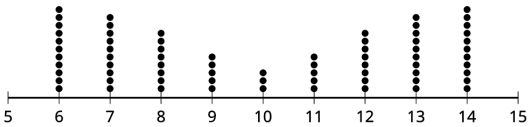
a.



b.

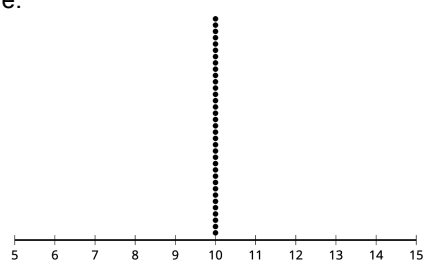


c.



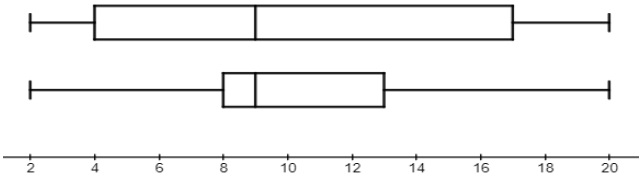
d.

e.

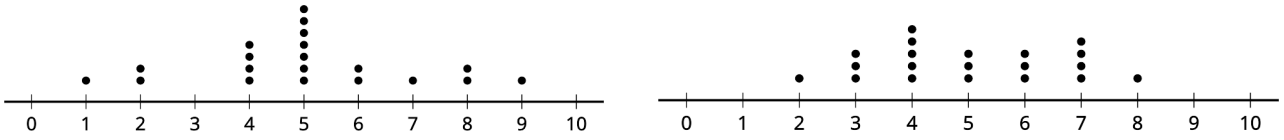


Are You Ready For More?

1. These two box plots have the same median and the same maximum and minimum values. How could we compare the variability of the two distributions?



2. These two dot plots have the same mean and the same standard deviation. How could we compare the variability of the two distributions?



Step 2

The purpose of this discussion is to make the connection between the shape of the distribution and the use of either IQR or standard deviation to quantify variability. Another goal is to make sure students understand that a greater value from IQR or standard deviation means greater variability.

Display the box plots in order of variability with the IQR included, and then display the dots plots in order of variability with the standard deviation included.

The IQR for the data in box plot distributions a through e are {40, 60, 50, 40, 20} and the standard deviation for the dot plot data in distributions a through e are approximately {2, 1.61, 2.92, 2.58, 0}. Here are some questions for discussion:

- "What are the IQR and standard deviation measuring?" (They are measuring the spread or variability of the data)
- "Which plots were the most difficult to arrange?" (The dot plots were more difficult because it was easy to find the IQR for the box plots.)
- "Do the orders given by the IQR and standard deviation match your order?" (Yes, except for the box plots a and d which had the same IQR and I didn't know how to arrange them.)
- "What do you notice about the values for IQR and standard deviation?" (The values for the standard deviation were higher than I thought except for distribution e. I did not know that the standard deviation could be equal to zero.)
- "What advantages are offered by using IQR and standard deviation versus visual inspection?" (The IQR and standard deviation are values that can be easily sorted.)

If some students already arranged the plots using IQR or standard deviation you should ask them, "Why did you choose to arrange the plots by IQR or standard deviation?" (I knew that IQR and standard deviation were measures of variability so I used them.)

RESPONSIVE STRATEGY

Provide students with a Venn diagram to compare how IQR and standard deviation are the same and different.

Supports accessibility for: Language; Organization

Use the *Collect and Display* routine as students are working by circulating and listening to students talk about the connection between the shapes of the distribution and the use of either the IQR or standard deviation to quantify variability. After arranging the box plots and dot plots in order of variability, add common or important phrases you hear students say about each representation to the visual display of student language from the previous activity. Capture words such as "greater variability," "skewed," or "spread of data." This will help students read and use mathematical language during their paired and whole-class discussions.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

In this lesson, students investigated variability using data displays and summary statistics. Facilitate a discussion using these questions.

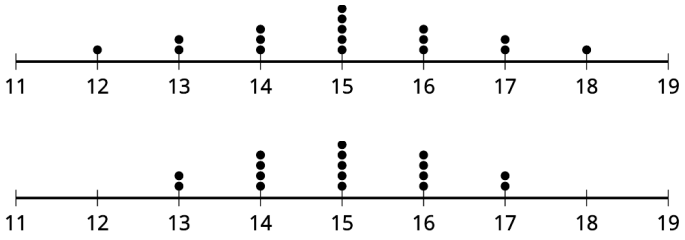
Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "One data set's measure of center is best represented by a median of 7 and another data set by a median of 10. How would you determine which data set has greater variability?" (You calculate the IQR. Whichever one has a larger IQR is more variable.)
- "How do you determine which of two roughly symmetric distributions has less variability?" (You calculate the standard deviation. Whichever one has a smaller standard deviation has less variability.)
- "What does it mean to say that one data set or distribution has more variability than another?" (The appropriate measure of variability for one data set is greater than the other. Using a data display, one distribution is more spread apart than the other.)

PLANNING NOTES

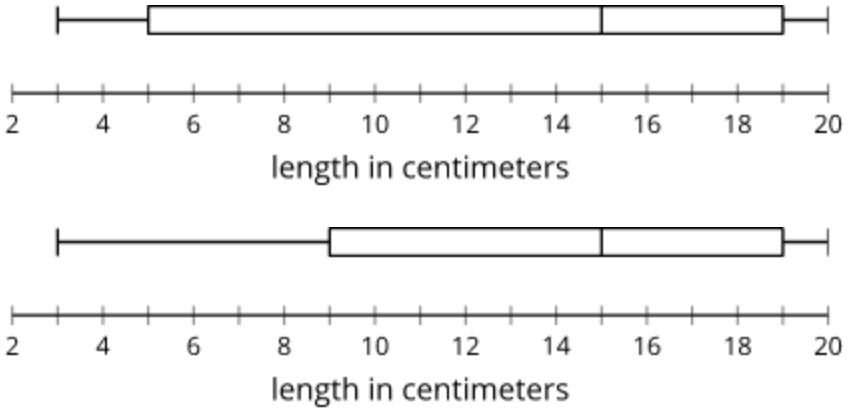
Student Lesson Summary and Glossary

Here are two dot plots, each with a mean of 15 centimeters, displaying the length of sea scallop shells in centimeters.



Notice that both dot plots show a symmetric distribution, so the mean and the standard deviation are appropriate choices for describing center and variability. The data in the first dot plot appear to be more spread apart than the data in the second dot plot, so you can say that the first data set appears to have greater variability than the second data set. This is confirmed by the standard deviation. The standard deviation of the first data set is 1.53cm and the standard deviation of the second data set is approximately 1.19cm. The greater the standard deviation of the data, the greater the variability of the data.

These two box plots represent the distributions of the lengths in centimeters of a different group of sea scallop shells, each with a median of 15 centimeters.



Notice that neither of the box plots have a symmetric distribution. The median and the IQR are appropriate choices for describing center and variability for these data sets. The data points in the middle half of the first box plot appear to be more spread apart, or show greater variability, than data points in the middle half of the second box plot. The IQR of the first distribution is 14 cm and 10 cm for the second data set. The IQR measures the difference between the median of the second half of the data, Q3, and the median of the first half, Q1, of the data, so it is not impacted by the minimum or the maximum value in the data set. It is a measure of the spread of the middle 50% of the data.

The standard deviation is calculated using every value in the data set, while the IQR is calculated using only the values for Q1 and Q3.

Cool-down: Which Menu? (*5 minutes*)

Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

Cool-down Guidance: More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

Cool-down

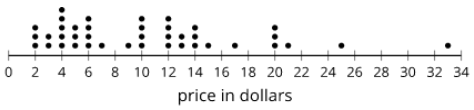
A restaurant owner believes that it is beneficial to have different menu items with a lot of variability in price so that people can have a choice of expensive and inexpensive food. Several chefs offer menus and suggested prices for the food they create. The owner creates dot plots for the prices of the menu items and finds some summary statistics.

Which menu best matches what the restaurant is looking for? Explain your reasoning.

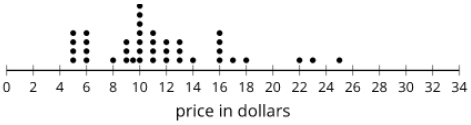
Italian:
- mean: $9.03
- median: $9
- standard deviation: $3.04
- IQR: $3.50

Diner:
- mean: $3.24
- median: $2
- standard deviation: $3.02
- IQR: $4

Japanese:
- mean: $10.35
- median: $10
- standard deviation: $6.97
- IQR: $9.50



Steakhouse:
- mean: $11.51
- median: $10.50
- standard deviation: $4.83
- IQR: $4.50



Student Reflection:

How do you feel about the way you participated in class today? How might you want to improve your participation in future lessons?

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure in the next lesson to ensure each student's ideas are a part of the collective learning?

Practice Problems

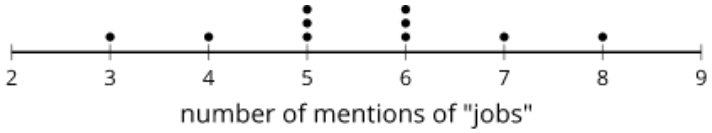
1. In science class, Clare and Lin estimate the mass of eight different objects that actually weigh 2,000 grams each. Some summary statistics for each set of eight measurements:

   Clare
   - mean: 2,000 grams
   - standard deviation: 275 grams
   - median: 2,000 grams
   - IQR: 500 grams

   Lin
   - mean: 2,000 grams
   - standard deviation: 225 grams
   - median: 1,950 grams
   - IQR: 350 grams

   Which student was better at estimating the mass of the objects? Explain your reasoning.

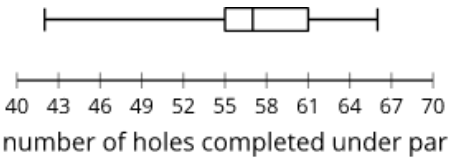
2. A reporter counts the number of times a politician talks about jobs in their campaign speeches. What is the standard deviation of the data represented in the dot plot?



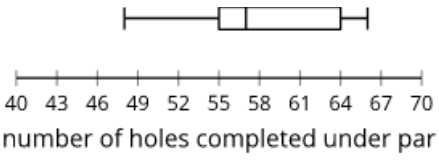
   a. 1.4 mentions
   b. 2.1 mentions
   c. 2.5 mentions
   d. 5.5 mentions

3. Four amateur miniature golfers attempt to finish 100 holes of golf under par several times. “Under par” means it took them fewer strokes than what was expected for that hole. For each round of 100, the number of holes they successfully complete under par is recorded. Due to the presence of extreme values, box plots were determined to be the best representation for the data. List the four box plots in order of variability from least to greatest.

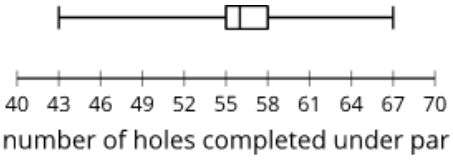
player a



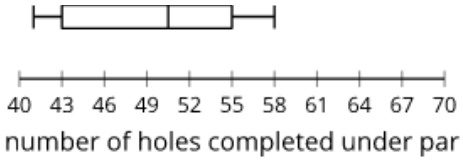
player b



player c



player d



4. Select **all** the distribution shapes for which the median *could be* much less than the mean.

   a. symmetric
   b. bell-shaped
   c. skewed left
   d. skewed right
   e. bimodal

   (From Unit 1, Lesson 11)

5.

a. What is the five-number summary for the data 0, 2, 2, 4, 5, 5, 5, 5, 7, 11?

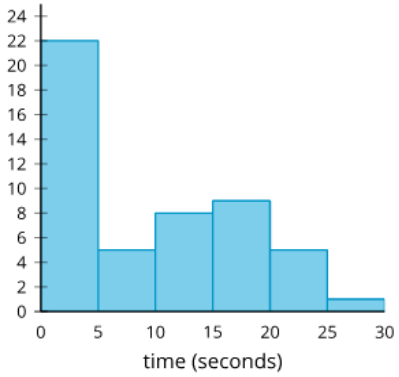
b. When the minimum, 0, is removed from the data set, what is the five-number summary?

(From Unit 1, Lesson 6)

6. What effect does eliminating the highest value, 180, from this data set have on the mean and median?
25, 50, 50, 60, 70, 85, 85, 90, 90, 180

(From Unit 1, Lesson 8)

7. The histogram represents the distribution of the number of seconds it took for each of 50 students to find the answer to a trivia question using the internet.



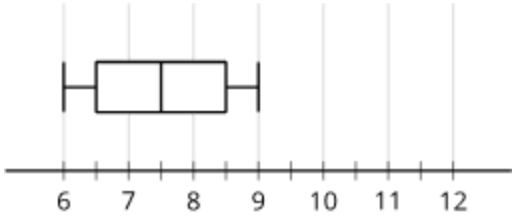
Which interval contains the median?

a. 0 to 5 seconds
b. 5 to 10 seconds
c. 10 to 15 seconds
d. 15 to 20 seconds

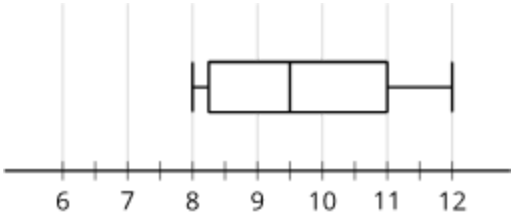
(From Unit 1, Lesson 6)

8. Here are the shoe sizes from two cohorts in the military.

Cohort A



Cohort B



a. Which cohort has more variability?

b. Does at least one person from cohort A have a bigger shoe size than someone from cohort B? Explain your reasoning.

c. Compare the measures of center.

(Addressing NC.6.SP.3b)

Lesson 14: Interpreting Data Sets

PREPARATION

Lesson Goals	Learning Targets
• Describe (orally and in writing) the meaning of measures of center and variability in context. • Describe (orally and in writing) how outliers impact measures of center and measures of variability.	• I can compare two data sets using the measures of center and variability. • I can tell how an outlier will impact mean, median, IQR, or standard deviation.

Lesson Narrative

The purpose of this lesson is to compare data sets using the measure of center (mean or median) and the measure of variability (standard deviation or IQR). First, students will be given various situations with two different data displays and their corresponding statistics. They will use the information provided to compare the two data sets and interpret the differences within the context of the problem.

Students will also recognize the impact of **outliers** on measures of center and measures of variability. They will explore these measures when there is an outlier included in the data set and when it is removed. This connects to their previous work on the effect of extreme values on the measures of center.

Interpreting measures of center and measures of variability in context is evidence that students are reasoning abstractly and quantitatively (MP2).

What do you hope to learn mathematically from this lesson?

Focus and Coherence

Building On	Addressing
NC.7.SP.3: Recognize the role of variability when comparing two populations. **a**. Calculate the measure of variability of a data set and understand that it describes how the values of the data set vary with a single number. • Understand the mean absolute deviation of a data set is a measure of variability that describes the average distance that points within a data set are from the mean of the data set. • Understand that the range describes the spread of the entire data set. • Understand that the interquartile range describes the spread of the middle 50% of the data **NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots).	**NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets. **NC.M1.S-ID.3:** Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Activity 1** (*10 minutes*)
  - Statistical technology is required for this lesson: Acquire devices that can access Desmos (recommended) or other spreadsheet technology. It is ideal if each student has their own device. The technology will be used for calculating measures of center (mean and median), measures of variability (standard deviation and IQR), and creating data displays (histograms and box plots).
- **Activity 2** (*15 minutes*)
  - Data set: http://bit.ly/L14dataset
- **Lesson Debrief** (*10 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L14 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

Building On: NC.7.SP.3a

This bridge supports students by allowing them to practice using measures of center and measures of variability to compare data sets. Students look at the pair of data sets and choose the better scenario based on the variability of each. In comparing the data sets, students will apply statistics to real-world scenarios.This task prepares students to make comparisons of data later in this lesson.

Student Task Statement

A health inspector for the county is trying to decide which restaurant to go to for their final inspection of the week. Here are each restaurant's health inspection ratings over the past year. Based on the restaurants' ratings, which restaurant should the inspector go to? Use the median and interquartile range to support your choice.

- Restaurant A: 88, 87, 89, 90, 87, 85, 88, 91, 86, 86, 88, 89
- Restaurant B: 90, 65, 89, 50, 94, 93, 95, 95, 75, 70, 88, 89

DO THE MATH

PLANNING NOTES

Activity 1: Interpreting Differences Between Distributions (*10 minutes*)

Instructional Routine: Discussion Supports (MLR8)	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

The mathematical purpose of this activity is to interpret the measures of center and measures of variability and use the measures to compare data sets in the context of the problem.

Step 1

- Ask students to form pairs or use visibly random grouping.
- Provide students 5 minutes to work on the questions.

***Advancing Student Thinking*:** Identify students who struggle making the connections between the summary statistics and the problem context. Ask them what information the statistics convey. For example, the mean or median represents a typical value. Tell them to focus less on the actual value of the standard deviation and IQR and more on which of the two distributions have a greater standard deviation or IQR and what that might mean.

RESPONSIVE STRATEGY

To help students get started, display sentence frames such as, "In this situation, a larger/smaller value for the standard deviation means ____." or "The medians/means are different by...."

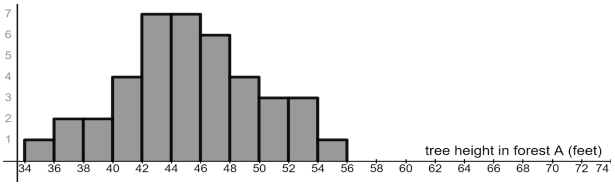
Supports accessibility for: Language; Organization

Student Task Statement

1. The heights of the 40 trees in each of two forests are collected.

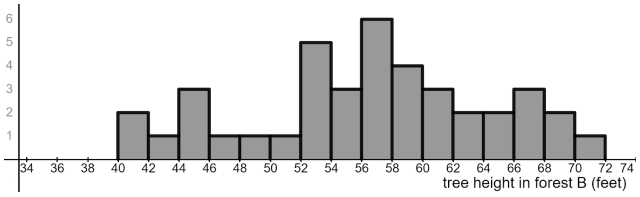
Forest A

mean: 44.8 feet, standard deviation: 4.72 feet



Forest B

mean: 56.03 feet, standard deviation: 7.87 feet



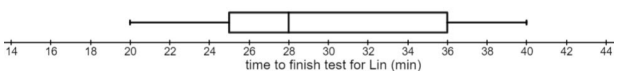
   a. Interpret the measure of center in terms of the situation.

   b. Interpret the measure of variability in terms of the situation.

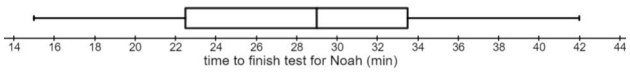
   c. Use these two measures to compare the two data sets.

2. The number of minutes it takes Lin and Noah to finish their tests in German class is collected for the year.

Lin
median: 28 minutes, IQR: 11 minutes



Noah
median: 29 minutes, IQR: 11 minutes



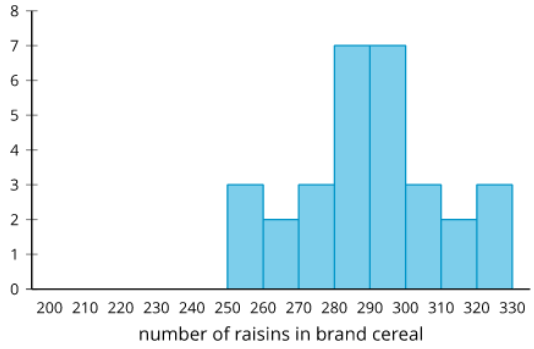
   a. Interpret the measure of center in terms of the situation.

   b. Interpret the measure of variability in terms of the situation.

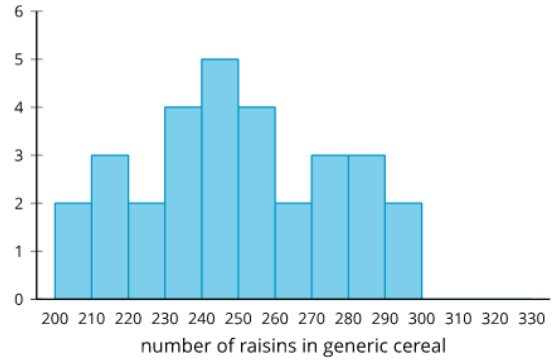
   c. Compare the two data sets.

3. The number of raisins in a cereal with a name brand and the generic version of the same cereal are collected for several boxes.

Brand cereal
mean: 289.1 raisins, standard deviation: 19.8 raisins



Generic cereal
mean: 249.2 raisins, standard deviation: 26.4 raisins



a. Interpret the measure of center in terms of the situation.

b. Interpret the measure of variability in terms of the situation.

c. Compare the two data sets.

Are You Ready For More?

Standard deviation gives a natural scale as to how far above or below the mean of a data point is. This is incredibly useful for comparing points from two different distributions.

For example, they say you cannot compare apples and oranges, but here is a way. The average weight of a granny smith apple is 128 grams with a standard deviation of about 10 grams. The average weight of a navel orange is 140 grams with a standard deviation of about 14 grams. If we have a 148-gram granny smith apple and a 161-gram navel orange, we might wonder which is larger for its species even though they are both about 20 grams above their respective mean. We could say that the apple, which is 2 standard deviations above its mean, is larger for its species than the orange, which is only 1.5 standard deviations above its mean.

1. The tallest tree in forest A is 55 feet. How many standard deviations above the mean height is it?
2. The tallest tree in forest B is 70 feet. How many standard deviations above the mean height is it?
3. Which tree is taller in its forest?

Step 2

- Facilitate a whole-class discussion. The purpose of this discussion is for students to interpret the measures of center and measures of variability, and use the measures to compare data sets in the context of the problem.
- For each set of graphs, select students to read their answers for all three prompts. When necessary, prompt students to revise their language to include the terms shape, measure of center, and variability.
- Use *Discussion Supports* as students share their observations. For each observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language (shape, measure of center, and variability). Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking and to clarify, if necessary. Call students' attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they explain the measures of center and measures of variability in context.

Why This Routine? *Discussion Supports* (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use *Discussion Supports* to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

DO THE MATH

PLANNING NOTES

Activity 2: Investigating Outliers (*15 minutes*)

Instructional Routines: Co-Craft Questions (MLR5); Collect and Display (MLR2)	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2; NC.M1.S-ID.3

The mathematical purpose of this activity is for students to investigate the impact of outliers on measures of center and variability, and to make decisions about whether or not to include outliers in a data set. Students will compare the statistical measures of the data set when the outlier is included to the same measures when the outlier is removed.

This may be the first time in the course that students participate in a *Co-Craft Questions* routine.

CO-CRAFT QUESTIONS

What Is This Routine? Students are presented with a picture, video, diagram, data display, or description of a situation, and their job is to generate one or more mathematical questions that could be asked about the situation. Students then share and compare their questions, as the teacher calls attention to questions that align with the content goals of the lesson. Finally, the "official" question or problem is revealed for students to work on.

Why This Routine? *Co-Craft Questions* (MLR5) allows students to get inside of a context before feeling pressure to produce answers, and creates space for students to produce the language of mathematical questions themselves. Use this routine to spark curiosity about a new mathematical idea or representation, and to elicit everyday student language to brainstorm about the quantitative relationships that might be investigated. During this routine, students use conversation skills and develop meta-awareness of the language used in mathematical questions and problems.

Step 1

- Provide access to devices that can run Desmos or other statistical technology.
- Display the data showing Per Capita Health Spending by Country in 2019 for all to see. Ensure that students understand what "per capita" means. "Per capita health spending" means the average health spending per person. For example, in 2019, approximately $11,072 was spent on healthcare for each person in the population in the United States.

RESPONSIVE STRATEGY

Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time.

Supports accessibility for: Organization; Attention

- Before asking students to answer the given questions, use the *Co-Craft Questions* routine to elicit questions that could be asked about the data.
  - Display only the data and ask students to write down possible mathematical questions about this data distribution. Keep in mind students do not need to answer their created questions.
  - Invite students to share and revise their questions with a partner and then with the whole class.
  - Record questions shared with the class in a public space. This helps students produce the language of mathematical questions as they begin to reason about the relationship between extreme values and the measures of center and variability.

***Advancing Student Thinking*:** Students may have forgotten how to use the technology to complete all of the calculations. Remind them of the tool and the function commands for each of the measures.

Student Task Statement

This data set is the average amount of money, in thousands of dollars, per year, spent on each person in the country (per capita spending) for health care in 43 countries in 2019. This data can also be found here: http://bit.ly/L14dataset. Use this data for questions 1–3.

1.154	1.213	1.337	1.907	1.973	1.996	2.008	2.159
2.222	2.230	2.354	2.384	2.579	2.638	2.790	2.932
3.224	3.379	3.384	3.428	3.616	3.649	4.204	4.224
4.262	4.578	4.653	4.811	4.823	5.187	5.276	5.376
5.418	5.428	5.558	5.568	5.765	5.782	5.851	6.646
6.647	7.732	11.072					

Source: OECD (2021), Health spending (indicator).

1. One value in the set is an outlier. Which one is it? How do you know?
2. Use technology to create a histogram and a box plot of the data.
   a. How does the outlier appear in the histogram?
   b. How does the outlier appear in the box plot?
3. Use technology to find the mean, standard deviation, median, and IQR.

   The maximum value in this data set represents the spending for the United States. Although outliers should not be removed without considering their cause, it is important to see how influential outliers can be for various statistics.
4. Remove the value for the United States from the data set.
   a. Use technology to calculate the new mean, standard deviation, median, and IQR.
   b. How do the mean, standard deviation, median, and IQR of the data set with the outlier removed compare to those of the original data set?
5. A statistician is curious why the United States is an outlier in relation to these other countries. What are some questions he could research to learn more?

Step 2

- Before the whole-class discussion, give students the opportunity to talk with their partner about using statistical tools to calculate and display numeric statistics.

  - Using the *Collect and Display* routine, write down common or important phrases you hear students say about each data set, especially phrases that include words such as mean, median, outliers, standard deviation, or IQR. Write the students' phrases and words on a visual display.

- Continue to add to this display throughout the rest of the lesson. This will help students read and use mathematical language during their paired and whole-class discussion while making decisions about whether or not to include the outliers in a data set.

DO THE MATH

PLANNING NOTES

Lesson Debrief (*10 minutes*)

In this lesson, students compared data sets by interpreting differences in the measures of center (mean or median) and measures of variability (standard deviation or IQR). Here are some questions for reflection and discussion that build from the work of Activity 2. The goal is to make sure that students understand that outliers can significantly impact measures of center and variability.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "Which measure of center is more greatly affected by the inclusion of extreme values, the mean or median? Explain your reasoning." (The mean since it uses the actual numerical value rather than the position of the values like the median does.)
- "Which measure of variability is more greatly affected by the inclusion of extreme values, the standard deviation or the interquartile range? Explain your reasoning." (The standard deviation since it uses the mean as well as the numerical value of each number in the data set whereas the IQR only uses the position of the middle half of the data.)
- "What impact did the outlier have on the median, mean, standard deviation and IQR?" (The outlier caused the mean and standard deviation to be higher. Removing it lowered both measures.)
- "For the healthcare spending task, do you think that 9.8923 thousand, the per capita spending of the United States, should be eliminated from the data set? Why or why not?" (No. A data point should not be removed simply because it is an outlier. In this case, because of the outlier, it would be best to use the median and IQR for the summary statistics.)
- "What information does the measure of center tell you about the data?" (It is a single value used to represent a typical value from the data set.)

PLANNING NOTES

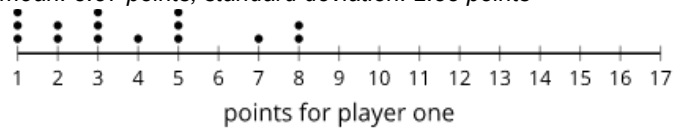
- "What information does the measure of variability tell you about the data?" (It represents how much the values in the data vary from one another. A large number means a lot of variation and a small number means there is little variation.)
- "How does an outlier affect the measures of center and the measures of variability?" (An outlier has the greatest impact on mean and standard deviation since both depend on all values in the data set. The median and interquartile range are more resistant to outliers.)

Student Lesson Summary and Glossary

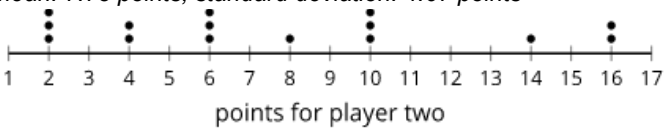
When comparing data sets it is useful to interpret the differences in the measure of center and measure of variability. Both the measure of center and the measure of variability have the same unit as the data set it describes. In the example below, the units are points.

The first dot plot shows the number of points that player one on a basketball team made during each of 15 games. The second dot plot shows the number of points scored by player two during the same 15 games.

mean: 3.87 points; standard deviation: 2.33 points



mean: 7.73 points; standard deviation: 4.67 points

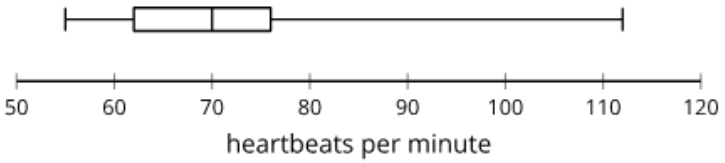


On average, player two scored 7.73 points in a game, about four more points than player one's average. Player one is more consistent in scoring her typical number of points. Player two has greater variability in points than player one because the data is more spread out. This is shown in the standard deviation for the second distribution being greater than the standard deviation for the first distribution.

Outlier: A data value that is unusual in that it differs quite a bit from the other values in the data set.

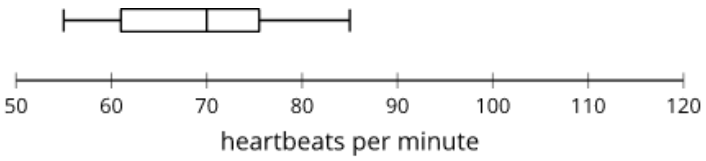
The box plot displays the resting heart rate, in beats per minute (bpm), of 50 athletes taken five minutes after a workout.

mean: 69.78 bpm
standard deviation: 10.71 bpm
minimum: 55 bpm
Q1: 62 bpm
median: 70 bpm
Q3: 76 bpm
maximum: 112 bpm



Suppose someone tells us that the maximum value of 112 bpm is an outlier. Once the outlier is removed, the box plot and summary statistics are:

mean: 68.92 bpm
standard deviation: 8.9 bpm
minimum: 55 bpm
Q1: 61 bpm
median: 70 bpm
Q3: 75.5 bpm
maximum: 85 bpm



The mean decreased by 0.86 bpm, and the median remained the same. The standard deviation decreased by 1.81 bpm. Based on the standard deviation, the data set with the outlier removed shows much less variability than the original data set containing the outlier. Since the mean and standard deviation use all of the numerical values, removing one very large data point can affect these statistics in important ways.

The median remained the same after the removal of the outlier and the IQR increased slightly. These measures of center and variability are much more resistant to change than the median and standard deviation. The median and IQR measure the middle of the data based on the number of values rather than the actual numerical values themselves, so the loss of a single value will not often have a great effect on these statistics.

Cool-down: Majors and Salaries (*5 minutes*)

Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2; NC.M1.S-ID.3
Cool-down Guidance: Points to Emphasize If students struggle with this, you may want to begin the next lesson looking at this cool-down and asking students what information they used to match the description to the data.	

Cool-down

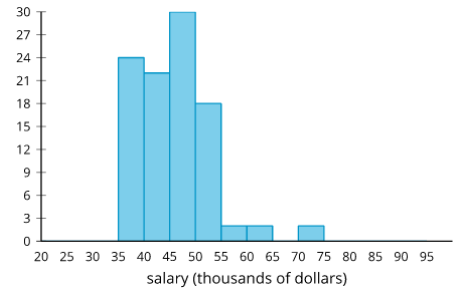
A college is looking at the data for its most recent college graduates based on their major.

- The mean salary of 100 recent college graduates who majored in engineering was $63,750 with a standard deviation of $10,020.
- The mean salary of 100 recent college graduates who majored in business was $52,200 with a standard deviation of $19,400.
- The mean salary of 100 recent college graduates who majored in the social sciences was $45,230 with a standard deviation of $6,750.

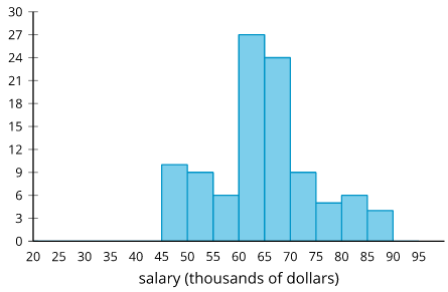
1. Match each histogram to the majors based on the description.

engineering business social sciences

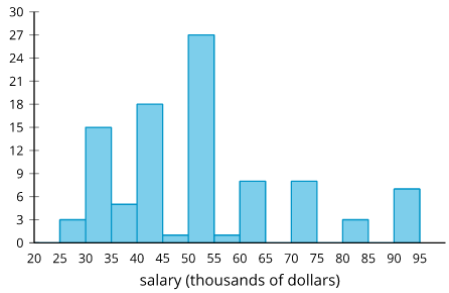
a.



b.



c.



2. For the histogram in problem 1a, a value of 70+ thousand was found to be misreported and was removed from the data set.
   a. Which measure of center (mean or median) and which measure of variability (standard deviation or IQR) was impacted the most by the removal of the value?
   b. Explain the impact removing the value has on these measures.

Student Reflection:

I am my most joyful in math when _____.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What opportunities are you giving students to reflect on their understanding of the mathematical content?

Practice Problems

1. Three drivers competed in the same fifteen drag races. The mean and standard deviation for the race times of each of the drivers are given.

   - Driver A had a mean race time of 4.01 seconds and a standard deviation of 0.05 seconds.
   - Driver B had a mean race time of 3.96 seconds and a standard deviation of 0.12 seconds.
   - Driver C had a mean race time of 3.99 seconds and a standard deviation of 0.19 seconds.

   a. Which driver had the fastest typical race time?

   b. Which driver's race times were the most variable?

   c. Which driver do you predict will win the next drag race? Support your prediction using the mean and standard deviation.

2. The number of letters received in the mail over the past week is recorded.

   2 3 5 5 5 15

   a. Which value appears to be an outlier?

   b. Explain what would happen to the mean and standard deviation if the 15 were removed from the data set.

3. A group of pennies made in 2018 are weighed. The mean is approximately 2.5 grams with a standard deviation of 0.02 grams. Interpret the mean and standard deviation in terms of the context.

4. The height of everyone in a fourth grade classroom, including the teacher, was collected. They had a mean height of 50 inches and a standard deviation of 4.4 inches.

   If the height of the teacher is removed, what effect might that have on the mean and standard deviation? Explain your reasoning.

5. The mean exam score for the first group of twenty examinees applying for a security job is 35.3 with a standard deviation of 3.6. The mean exam score for the second group of twenty examinees is 34.1 with a standard deviation of 0.5. Both distributions are close to symmetric in shape.

   a. Use the mean and standard deviation to compare the scores of the two groups.

   b. The minimum score required to get an in-person interview is 33. Which group do you think has more people get in-person interviews?

6. Andre records how long it takes him (in minutes) to hike a mountain each day for six days.

   50 52 58 55 59 50

   a. Use technology to calculate the mean number of minutes it takes Andre to hike a mountain.

   b. What do you think will happen to the mean time for the week if Andre decides to take pictures of the trees, waterfalls, and wildlife throughout the hike for the seventh day?

   c. Calculate the mean number of minutes including the time it took Andre on the seventh day: 50, 52, 58, 55, 59, 50, 130.

   d. If Andre didn't stop to take pictures on the seventh day, he thinks he could have finished the trail in 60 minutes. Calculate the mean hiking time using Andre's estimate for the seventh day: 50, 52, 58, 55, 59, 50, 60.

7. The number of different species of plants in some gardens is recorded.

   1 2 3 4 4 5 5 6 7 8

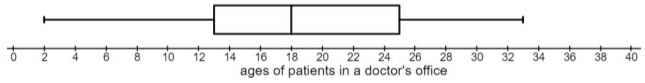
   a. What is the mean?

   b. What is the standard deviation?

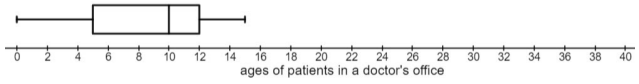
   (From Unit 1, Lesson 13)

8. List the four box plots in order of variability from least to greatest.

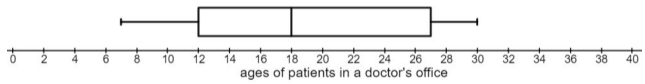
   a.

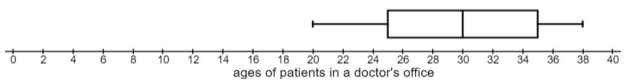
   b.

   c.

   d.

   (From Unit 1, Lesson 13)

9. At the end of last year, teachers were rated by their students on a 0–10 scale. Two of the teachers' ratings are given. Whose class would you register for? Explain your reasoning.

   a. Teacher A: 9, 8, 10, 10, 7, 1, 8, 1, 2, 8

   b. Teacher B: 9, 8, 8, 7, 9, 7, 7, 9, 7, 8

   (Addressing NC.7.SP.3.a)

Lesson 15: Comparing Data Sets

PREPARATION

Lesson Goal	Learning Target
• Compare and contrast (orally and in writing) situations using measures of center and measures of variability.	• I can compare and contrast situations using measures of center and measures of variability.

Lesson Narrative

This culminating lesson provides opportunities for students to collaborate, share mathematical ideas, and reflect on their mathematical thinking about all that they have learned in this unit. This lesson also gives students another opportunity to compare measures of center and variability, including using standard deviation and outliers in the comparisons. When students are describing measures of center and measures of variability in the context of marathon time, they are reasoning abstractly and quantitatively (MP2) because they are interpreting the meaning of their answer in context.

Share some ways you see this lesson connecting to previous lessons in this unit. What connections will you want to make explicit?

Focus and Coherence

Building On	Addressing
NC.6.SP.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **NC.M1.S-ID.1:** Use technology to represent data with plots on the real number line (histograms and box plots).	**NC.M1.S-ID.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
- **Activity 2** (*Optional, 20 minutes*)
- **Lesson Debrief** (*5 minutes*)
- **Cool-down** (*5 minutes*)
  - M1.U1.L15 Cool-down (print 1 copy per student)

LESSON

Bridge (*Optional, 5 minutes*)

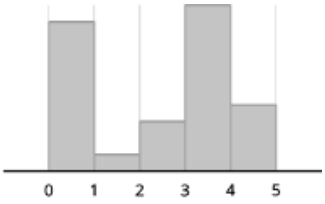
Building On: NC.6.SP.2	**Addressing:** NC.M1.S-ID.2

This bridge supports students in using the distribution shape to make comparisons between data displays. Students justify making a decision from a set of displays. This bridge is helpful for students who may have struggled with question 1 from Check Your Readiness and is connected to cool-downs from Lessons 7, 13, and 14, in which students must take distribution shape into account in order to answer questions about the data.

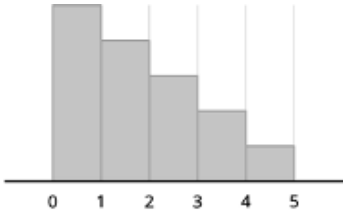
Student Task Statement

Each histogram represents the number of star ratings for a different restaurant. The ratings range from 0–4, with 0 representing a very poor experience and 4 representing an excellent experience.

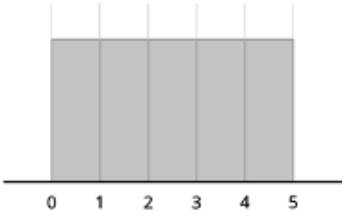
Restaurant A



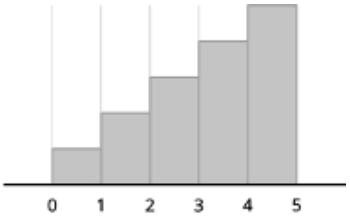
Restaurant B



Restaurant C



Restaurant D



Questions	Explain your reasoning
1. Which restaurant do reviewers like the most?	
2. Which restaurant do reviewers like the least?	
3. Which restaurant received mostly mixed reviews?	
4. Which restaurant would you choose to try?	

DO THE MATH

PLANNING NOTES

Warm-up: Bowling Partners (*10 minutes*)

Instructional Routine: Round Robin
Building Towards: NC.M1.S-ID.2

The mathematical purpose of this activity is for students to compare different distributions using shape, measures of center, and measures of variability. This warm-up prompts students to compare four distributions representing recent bowling scores for potential teammates. It gives students a reason to use language precisely (MP6).

Step 1

- Ask students to arrange themselves in small groups or use visibly random grouping.
- Encourage students to work independently for 2 minutes and then come to consensus with their groups by sharing their reasoning using the *Round Robin* routine.

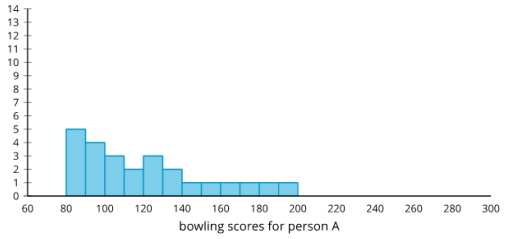
Why This Routine? Engaging a group of students in collaborative problem solving, with equitable inclusion of ideas, can be challenging due to normative social status issues that place higher value on some students' contributions over others. *Round Robin* allows all students to include their rough draft ideas for solving an open-ended problem without a subset of students dominating the conversation. Knowing all ideas will be shared should motivate all students to try at least one strategy to solve a problem on their own, critical for making sense of problems and persevering in solving them (MP1). The active sharing and listening involved in this routine also provides opportunity for constructing and critiquing viable arguments (MP3).

Monitoring Tip: While students are working in small groups, listen for how they use terminology and talk about characteristics of the images in comparison to one another.

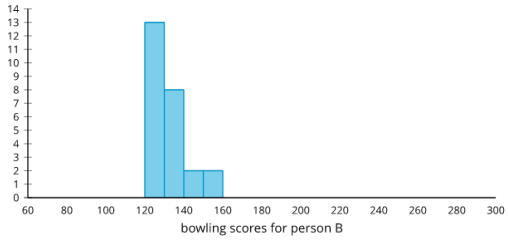
Student Task Statement

Each histogram shows the bowling scores for the last 25 games played by each person. Choose two of these people to join your bowling team. Explain your reasoning.

Person A
- mean: 118.96
- median: 111
- standard deviation: 32.96
- interquartile range: 44

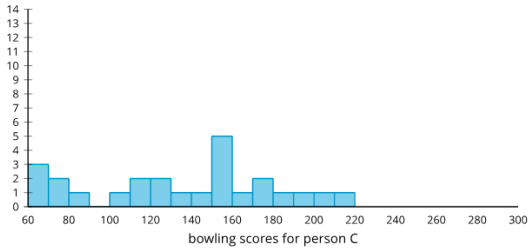


Person B
- mean: 131.08
- median: 129
- standard deviation: 8.64
- interquartile range: 8



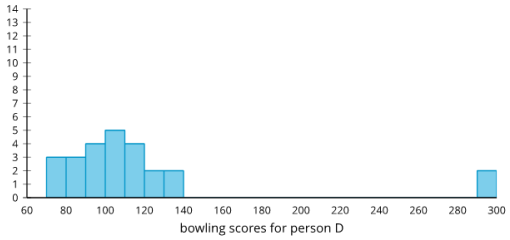
Person C

- mean: 133.92
- median: 145
- standard deviation: 45.04
- interquartile range: 74



Person D

- mean: 116.56
- median: 103
- standard deviation: 56.22
- interquartile range: 31.5



Step 2

- Remind students that there is not one correct answer and they should include how they used statistics and histograms in their selection before asking them to share their reasoning.
- In a whole-class discussion, ask each group to share one bowler they would choose and their reasoning. If none of the groups select a certain player, ask why this player was not chosen or why another team may want this player on their team.

DO THE MATH

PLANNING NOTES

Activity 1: Comparing Measures (*20 minutes*)

Instructional Routine: Take Turns	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

In this activity, students take turns with a partner determining the best measure of center and the best measure of variability for several data sets. Students trade roles explaining their thinking and listening, providing opportunities to explain their reasoning and critique the reasoning of others (MP3). Students also determine which data set has a greater measure of center and which has a greater measure of variability.

Step 1

- Ask students to form pairs or use visibly random grouping.
- If necessary, demonstrate the *Take Turns* routine before students start working.

  - Tell students that for each data display or description of a data set in column A, one partner determines the appropriate measure of center and measure of variability and explains why they think it is appropriate. The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement.
  - For the next data display or description of a data set in column B, the students swap roles.
- Problem 7 has a part c. Students can work together to determine the best measures for part c.
- Once an agreement is reached for each group of data sets, students will determine which data set has the greatest measure of center, and which data set has the greatest measure of variability.
  - Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, “That measure of center could/couldn’t be correct because….” or “Based on the shape of the distribution, a better choice would be _____ because….”
  - Note that in problem 6, it is not clear which measure of center would be greater. Students can argue either way.

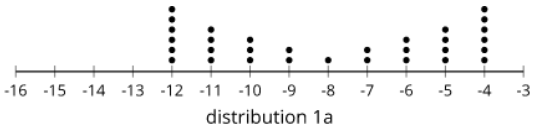
***Advancing Student Thinking*:** For the situations described in words, students may think there is not enough information to answer the question. Ask these students, "What do you think the distributions might look like for the situations described?" Tell them to use their distributions to answer the question and be prepared to explain their reasoning.

Student Task Statement

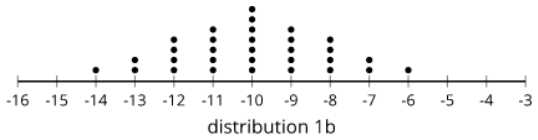
For each group of data sets:

- Determine the best measure of center and measure of variability to use based on the shape of the distribution.
- Determine which set has the greatest measure of center.
- Determine which set has the greatest measure of variability.
- Be prepared to explain your reasoning.

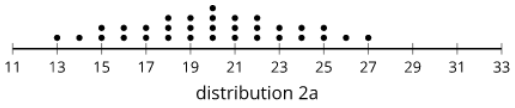
1a



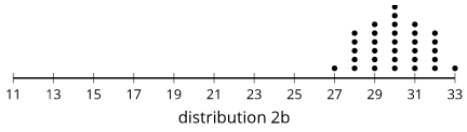
1b



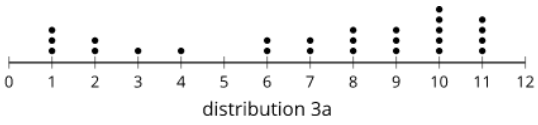
2a



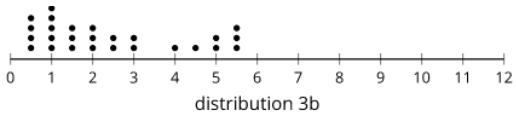
2b



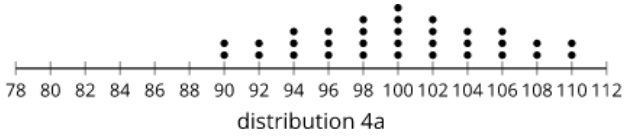
3a



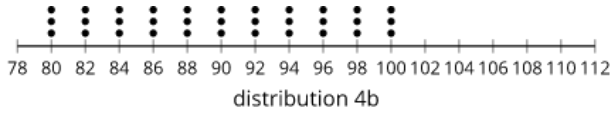
3b



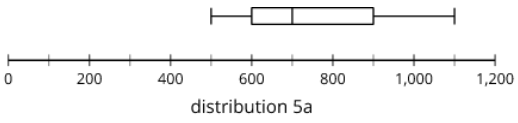
4a



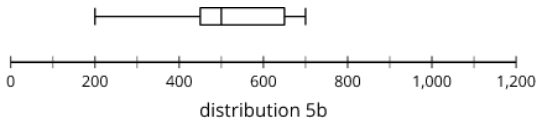
4b



5a



5b



6a
A political podcast has mostly reviews that either love the podcast or hate it.

6b
A cooking podcast has reviews that neither hate nor love the podcast.

7a Stress testing concrete from site A has all 12 samples break at 450 pounds per square inch (psi).

7b Stress testing concrete from site B has samples break every 10 psi starting at 450 psi until the last core is broken at 560 psi.

7c Stress testing concrete from site C has 6 samples break at 430 psi and the other 6 break at 460 psi.

Step 2

- Select students to share how they determined whether to use the mean or the median and how they figured out which data set showed greater variability.

DO THE MATH

PLANNING NOTES

Activity 2: Comparing Marathon Times (*Optional, 20 minutes*)

Instructional Routines: Aspects of Mathematical Modeling; Co-Craft Questions (MLR5)	
Building On: NC.M1.S-ID.1	**Addressing:** NC.M1.S-ID.2

The mathematical purpose of this activity is for students to compare measures of center and measures of variability in context.

Monitoring Tip: Monitor for students who:
- determine the slower age group by using an informal description of the shift in data
- determine the slower age group by using a numerical estimate for the mean or median for measures of center
- determine variability from the range of values
- use a numerical estimate for IQR or standard deviation as measures of variability

Step 1

- Provide time for students to use technology to engage in *Aspects of Mathematical Modeling* by researching marathons to answer question 1. Some students may get initially frustrated at not being able to determine a “right” answer for part c. Encourage them to list their own assumptions to hone in on a reasonable response.

Why This Routine? Mathematical modeling is often new territory for both students and teachers. Activities tagged as *Aspects of Mathematical Modeling* offer opportunities to develop discrete skills in the supported environment of a classroom lesson to make success more likely when students engage in more open-ended modeling.

Student Task Statement

1. Do some research on marathons to answer the following questions:
   a. How long is a marathon?
   b. Name a place that is approximately that same distance from where you live or go to school.
   c. How long does it typically take someone to finish a marathon?
      i. What factors play into how long it takes someone?
      ii. Name some assumptions you made to determine that “typical” time.

2. Here are the marathon finishing times for two different age groups, in minutes. What are some possible mathematical questions that could be asked about these data sets?
   a. Ages 30–39 {232, 238, 240, 243, 243, 245, 251, 252, 258, 259, 260, 262, 263, 265, 265, 265, 270, 278, 279, 283, 284, 291, 291, 293, 293, 300, 305, 310, 312, 319, 323, 324, 376, 377, 379}
   b. Ages 40–49 {250, 260, 262, 262, 263, 265, 271, 272,272, 283, 284, 298, 298, 299, 299, 300, 302, 304, 310, 319, 320, 321, 322, 324, 338, 338, 342, 343, 348, 348, 362, 363, 379, 380, 382, 385, 442, 448, 450}

Step 2

- Display the marathon finishing times for the two different age groups and use the *Co-Craft Questions* routine to invite pairs of students to write possible mathematical questions about the situation.

  - Invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the relationships between the two data sets in this task, the measures of center and measures of variability.

Step 3

- Students choose to work individually or in pairs to continue the activity.

Student Task Statement

3. Use technology to create a data display for each age group of runners and then describe the distribution for each.
4. Which age group tends to take longer to run the marathon? Explain your reasoning.
5. Which age group shows more variability in its finishing times? Explain your reasoning.

Are You Ready For More?

1. How do you think finish times for a 20–29 age range will compare to these two distributions?
2. Find some actual marathon times for this group and create a data display using technology to help compare.

Step 4

- Select students to share their answers and reasoning in a sequence that moves from less formal reasoning to more definite values and statistics.
- Ask students:
  - "What measure of center and measure of variability are most appropriate to use with the distributions? Explain your reasoning." (The median and IQR are the most appropriate because the data set is skewed (or because the data set has outliers)
  - "Which points show values that are most likely to be outliers?" (The slowest runners in each group, on the right end of the dot plot, might be outliers.)
  - "Based on the displayed information, are there any outliers in these data sets?" (No, there are no data points that appear to be extreme values (or are very removed from the data set) for either the 30–39 age group or the 40–49 age group.)

DO THE MATH

PLANNING NOTES

Lesson Debrief (*5 minutes*)

Facilitate a discussion to ensure students know how to compare data sets using measures of variability, including standard deviation, and measures of center.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

- "How do you compare the measures of variability for a data set?" (You either calculate them or estimate them from a data display.)
- "How do you estimate variability when looking at data displays?" (You try to estimate the center and then estimate how spread apart the data are.)
- "How do you determine which measure of center to use for a data set?" (You look at the shape and use the mean when it is symmetric or really close and the median when it is skewed or if there are outliers.)
- "Why is the median the preferred measure of center for skewed distributions?" (The median is preferred because it more accurately represents the center of the data. Data values farther from the center impact the median less than the mean, so the median remains near the typical values.)
- "Why is the mean the preferred measure of center for symmetric distributions?" (The mean is preferred because it takes into account all of the values in the data set when it is calculated.)

PLANNING NOTES

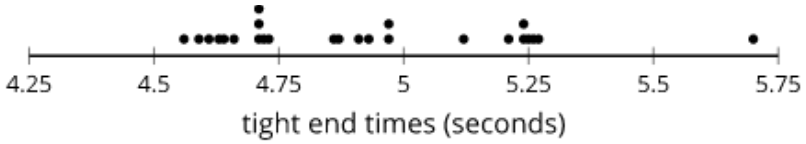
Student Lesson Summary and Glossary

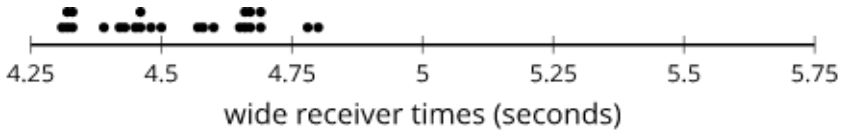
To compare data sets, it is helpful to look at the measures of center and measures of variability. The shape of the distribution can help choose the most useful measure of center and measure of variability.

When distributions are symmetric or approximately symmetric, the mean is the preferred measure of center and should be paired with the standard deviation as the preferred measure of variability. When distributions are skewed or when outliers are present, the median is usually a better measure of center and should be paired with the interquartile range (IQR) as the preferred measure of variability.

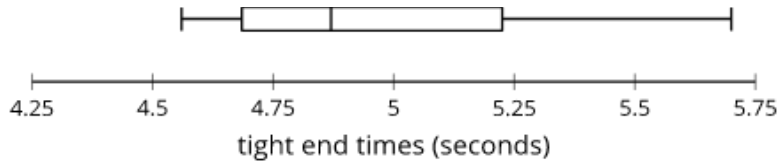
Once the appropriate measure of center and measure of variability are selected, these measures can be compared for data sets with similar shapes.

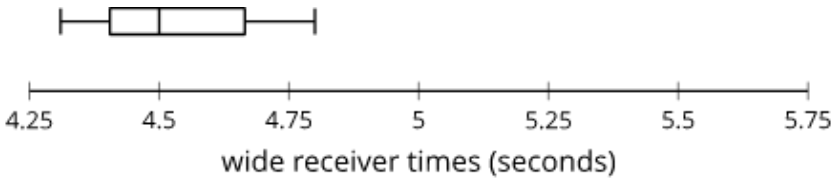
For example, let's compare the number of seconds it takes football players to complete a 40-yard dash at two different positions. First, we can look at a dot plot of the data to see that the tight end times do not seem symmetric, so we should probably find the median and IQR for both sets of data to compare information.





The median and IQR could be computed from the values, but can also be determined from a box plot.





This shows that the tight end times have a greater median (about 4.9 seconds) compared to the median of wide receiver times (about 4.5 seconds). The IQR is also greater for the tight end times (about 0.5 seconds) compared to the IQR for the wide receiver times (about 0.25 seconds).

This means that the tight ends tend to be slower in the 40-yard dash when compared to the wide receivers. The tight ends also have greater variability in their times. Together, this can be taken to mean that, in general, a typical wide receiver is faster than a typical tight end, and the wide receivers tend to have more similar times to one another than the tight ends do to one another.

Cool-down: Comparing Mascots (*5 minutes*)

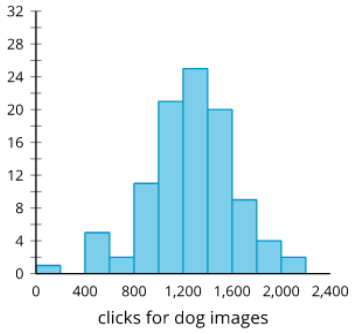
Addressing: NC.M1.S-ID.2

Cool-down Guidance: Points to Emphasize
If students struggle with either identifying the best measure of center to use based on the shape of data, or connecting mean to SD and median to IQR, invite selected students to share their data and discuss how they chose the appropriate measure of center and variability to describe their data set.

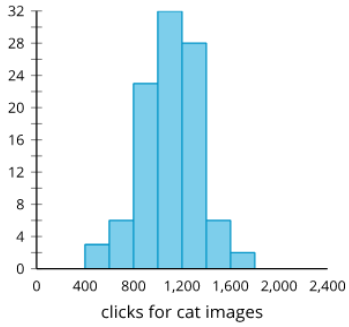
Cool-down

A new pet food company wants to sell their product online and use social media to promote themselves. To determine whether to use a dog or a cat as their mascot, they research the number of people who click on links with an image of a dog or a cat.

mean: 1,263.5 clicks
median: 1,282 clicks
standard deviation: 357.4 clicks
IQR: 409 clicks



mean: 1,105.4 clicks
median: 1,125.5 clicks
standard deviation: 239.3 clicks
IQR: 312.5 clicks



1. Based on the shape of the distributions, what measure of center and measure of variability would you use to compare the distributions? Explain your reasoning.

2. Based on the data shown here, should the company use a dog or cat mascot? Explain your reasoning.

Student Reflection:

Name something your math teacher can do to help you learn best.

DO THE MATH

INDIVIDUAL STUDENT DATA

SUMMARY DATA

NEXT STEPS

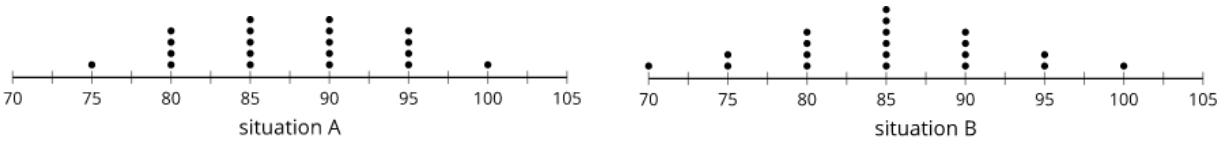
TEACHER REFLECTION

What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?

What question do you wish you had asked today? When and why should you have asked it?

Practice Problems

1. Twenty students participated in a psychology experiment that measured their heart rates in two different situations.



   a. What are the appropriate measures of center and variability to use with the data? Explain your reasoning.

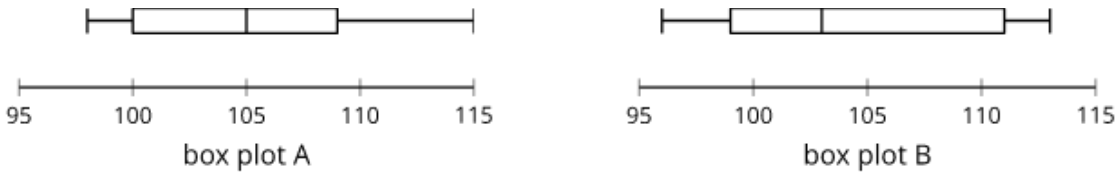
   b. Which situation shows a greater typical heart rate?

2.

   a. Invent two situations that you think would result in distributions with similar measures of variability. Explain your reasoning.

   b. Invent two situations that you think would result in distributions with different measures of variability. Explain your reasoning.

3. Here are box plots for two data sets:



   a. Which data set has a greater median?

   b. Which data set has a greater IQR?

4. The data set and some summary statistics are listed.

   11.5, 12.3, 13.5, 15.6, 16.7, 17.2, 18.4, 19, 19.5, 21.5

   mean: 16.52
   median: 16.95
   standard deviation: 3.11
   IQR: 5.5

   a. How does adding 5 to each of the values in the data set impact the shape of the distribution?

   b. How does adding 5 to each of the values in the data set impact the measures of center?

   c. How does adding 5 to each of the values in the data set impact the measures of variability?

   (From Unit 1, Lesson 6)

5. The depth of two lakes is measured at multiple spots. For the first lake, the mean depth is about 45 feet with a standard deviation of 8 feet. For the second lake, the mean depth is about 60 feet with a standard deviation of 27 feet.

   Noah says the second lake is generally deeper than the first lake. Do you agree with Noah?

   (From Unit 1, Lesson 14)

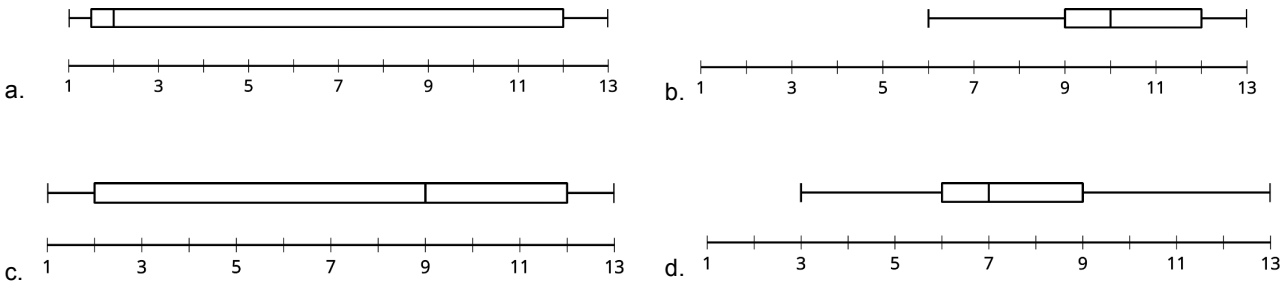
6. The dot plots display the height, rounded to the nearest foot, of maple trees from two different tree farms.

   a. Compare the mean and standard deviation of the two data sets.

   b. What does the standard deviation tell you about the trees at these farms?

   (From Unit 1, Lesson 13)

7. Which box plot has an IQR of 10?



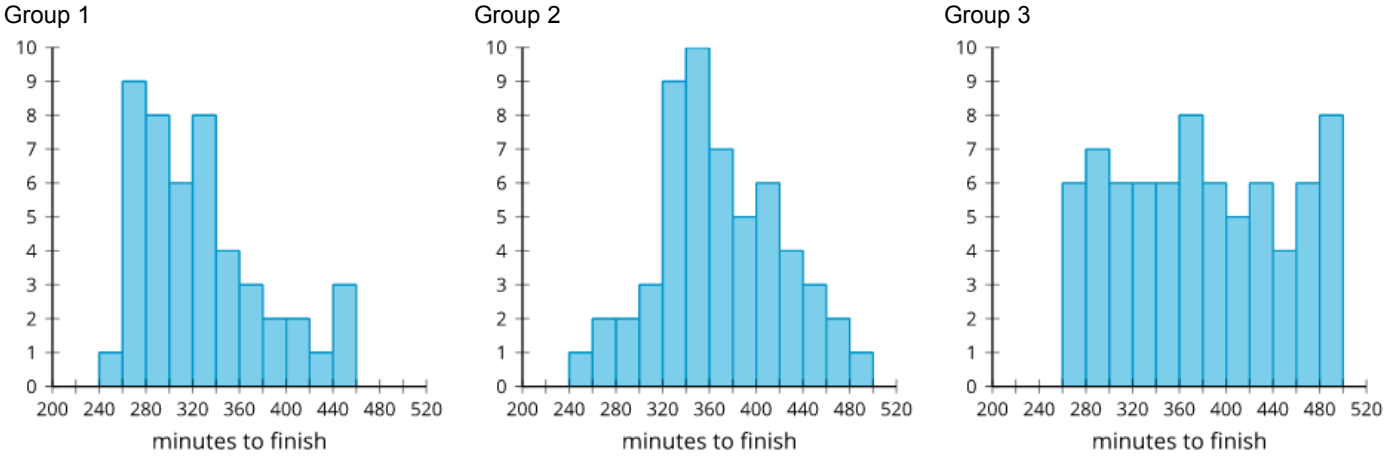
   (From Unit 1, Lesson 12)

8. What effect does eliminating the lowest value, -6, from the data set have on the mean and median?

   -6, 3, 3, 3, 3, 5, 6, 6, 8, 10

   (From Unit 1, Lesson 8)

9. These distributions represent marathon times for different groups.



   a. Which display is most likely to represent the marathon times for people aged 20–30? Explain your reasoning.

   b. Which display is most likely to represent the marathon times for every tenth person to cross the finish line? Explain your reasoning.

   c. Which display is most likely to represent the marathon times for people aged 40–50? Explain your reasoning.

   (Addressing NC.6.SP.2)

Lesson 16: Post-Test Activities

PREPARATION

Lesson Goals	Learning Targets
• Communicate high expectations for all students. • Build a welcoming classroom community that recognizes and values the unique perspectives and experiences each student brings.	• I understand the reasoning for and will strive to meet the expectations communicated by my teacher. • I know my classmates and can recognize the value I will add to this classroom community.

Lesson Narrative

This lesson, which should occur after the Unit 1 End-of-Unit Assessment, allows for students to reflect on the unit, share feedback, conference with the teacher, and engage in a fun community-building activity.

Gathering student feedback is a powerful and strategic way to learn about students and improve instructional practices. It also creates student and family buy-in and centers students as decision makers and problem solvers in their own learning.

Conducting one-on-one conferences with students, using the surveys as a data point, is encouraged. These conferences can be done as students are engaging in the Personality Coordinates Icebreaker. Potential conference topics include:

- student responses to the daily student reflections
- student response to the end of unit student survey (as students finish them)
- executive functioning skills
- student learning contracts
- goal setting and self-evaluation

What do you hope to learn about your students during this lesson?

Agenda, Materials, and Preparation

- **Activity 1** (*20 minutes*)
  - End-of-Unit 1 Student Survey (print 1 copy per student)
- **Activity 2** (*20 minutes*)
  - Personality Coordinates (print 1 copy per group of 4 students and cut in half in advance)

LESSON

Activity 1: End-of-Unit 1 Student Survey (*20 minutes*)

The End-of-Unit 1 Student Survey is a critical opportunity for teachers to gather low-stakes, non-evaluative feedback to support equity and instructional pedagogy. The survey is also highly beneficial for students as it is designed to encourage self-awareness, self-management, social awareness, relationship skills, and responsible decision making. Provide students a chance to quietly and independently complete this survey after they complete their testing.

Activity 2: Personality Coordinates Icebreaker[1] (*20 minutes*)

The purposes of this activity are to:

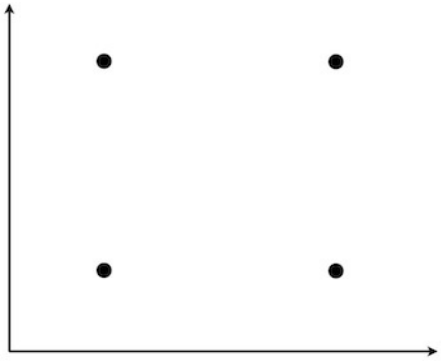
- Continue building a safe classroom community that recognizes what each student brings to the community.
- Informally remind students of the meaning of coordinate pairs as a bridge into Unit 2 (Linear Equations and Inequalities).

Step 1

- Students arrange themselves in groups of four or use visibly random grouping.
- Pass out one half-sheet to each group or have each group send their group facilitator to pick up a half-sheet.

Step 2

- Each person in a group picks a dot and writes their name next to it.
- The group determines the label for each axis that accurately describes each member of the group's place on the graph.



Note: Physical attributes don't require as much thought and don't reveal much new information about students, so discourage their use after the first round.

- Repeat, as time permits, with a new graph and groups changing members each time. Encourage students to join groups containing classmates they have not worked with yet.

[1] Adapted from Dan Meyer

TEACHER REFLECTION

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician throughout this work.

List ways you have seen yourself grow as a teacher.

What will you continue to do and what will you improve upon in Unit 2?